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A Matter of Perspective: Reliable Communication and Coping with Interference with Only Local Views

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Abstract

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This dissertation studies interference in wireless networks. Interference results from multiple simultaneous attempts to communicate, often between unassociated sources and receivers, preventing extensive coordination. Moreover, in practical wireless networks, learning network state is inherently expensive, and nodes often have *incomplete and mismatched views* of the network. The fundamental communication limits of a network with such views is unknown.

To address this, we present a local view model which captures asymmetries in node knowledge. Our local view model does not rely on accurate knowledge of an underlying probability distribution governing network state. Therefore, we can make robust statements about the fundamental limits of communication when the channel is quasi-static or the actual distribution of state is unknown: commonly faced scenarios in modern commercial networks. For each local view, channel state parameters are either perfectly known or completely unknown. While we propose no mechanism for network learning, a local view represents the result of some such mechanism.

We apply the local view model to study the two-user Gaussian interference channel: the smallest building block of any interference network.

All seven possible local views are studied, and we find that for five of the seven, there exists no policy or protocol that universally outperforms time-division multiplexing (TDM), justifying the orthogonalized approach of many deployed systems. For two of the seven views, TDM-beating performance is possible with use of opportunistic schemes where opportunities are revealed by the local view.

We then study how message cooperation — either at transmitters or receivers — increases capacity in the local view two-user Gaussian interference channel. The cooperative setup is particularly appropriate for modeling next-generation cellular networks, where costs to share message data among base stations is low relative to costs to learn channel coefficients. For the cooperative setting, we find: (1) opportunistic approaches are still needed to outperform TDM, but (2) opportunities are more abundant and revealed by more local views.

For all cases studied, we characterize the capacity region to within some known gap, enabling computation of the generalized degrees of freedom region, a visualization of spatial channel resource usage efficiency.

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Chapter 1

Introduction

One of the foremost challenges in current and future wireless networks is interference. Unlike other performance limiting phenomena such as thermal noise, low resolution quantizers, and processing speed — all of which may be addressed by future advances in circuitry and system design — interference is a product of immutable laws of electromagnetic propagation compounded by an ever-increasing consumer appetite for mobile data.

In theory, what should be done to optimally, or near-optimally, mitigate the effects of interference on network capacity is known for a variety of circumstances, including fast-fading ergodic channels, two-user channels with known state, and multiuser channels in the high signal-to-noise ratio regime. The first section of this chapter contains a full overview of these results.

Unfortunately, the key challenge in practice is knowing at any given moment which approach is optimal. Due to the time-varying nature of networks and the cost associated with learning about the network, it is rare that any wireless node has a complete and perfect understanding of network state. Therefore, any communication protocol designed for such a node necessarily must accommodate the differing "views" of nodes in the network.

This dissertation attacks fundamental analysis of interference networks from a slightly different perspective. The approach used has three key differentiating features from prior work:

Incomplete Mismatched Knowledge: We use a local view knowledge model (§2.2), which isolates challenges posed by incomplete knowledge of the network. Whereas the quality of parameter estimates may increase with improved circuitry — e.g., channel sounding has much lower mean squared error with reduced noise — the completeness of knowledge is dependent on the network architecture and management protocols that define whether each parameter is estimated or whether each estimate is forwarded to a particular node. Moreover, as relative position in the network varies from node to node, the incompleteness of knowledge is different for each node.

Non-Bayesian Uncertainty and Performance Metric: Our local view model is non-probabilistic. Many other approaches address uncertainty of unknowns by assuming a prior distribution on parameters in order to analyze the average performance over many realizations of the unknown. In certain scenarios the time scale of the variation of unknowns is much shorter than that of communication and assumed stationarity or ergodicity of the variation also leads to some notion of long term performance.

While appropriate for some cases, the Bayesian approach is decidedly not robust over short time-scales; only rarely does a short sequence empirically represent the typical characteristics of a distribution. Furthermore, blind assumption of an arbitrary prior may result in a mismatch between model and reality, leading to performance analysis with irrelevant predictive ability. In this respect, our non-probabilistic approach is more robust: we are able to provide performance guarantees for short time-scale communication demands (e.g., live streaming video), and we can comment on system performance even if the time-varying dynamics of the unknown parameters are poorly understood.

Distributed Notion of Capacity: Whereas it is common to consider the performance of a single fixed network state, optimization of a response to one arrangement may lead to poor performance in another. A classic example is the hidden-terminal problem in medium-access control [32]. This coupled with our non-Bayesian approach requires a crystallization of what it means for one protocol to be "better" than another, and what it means to be "optimal". In order to address this, we propose a distributed notion of channel capacity predicated upon a minimum-performance constraint (§2.5).

Generally speaking, we consider how well any protocol can perform, conditioned on the assumption that the protocol performs universally better than some reference. Within this thesis we choose time-division multiplexing (TDM) as a reference protocol primarily because it can be used with minimal knowledge. Thus, as local views become more complete, we are able to see how knowledge of a particular parameter enhances the fundamental limits on network performance.

With these three features differentiating our approach from prior work, we consider an often encountered practical scenario in interference networks as a first step towards bridging the gap between interference mitigation in theory and interference mitigation in practice.

1.1 State of the Art

Information theoretic study of interference began in the early 1960s [47]. In particular, the issue of the capacity region of the Gaussian interference channel (IC) was the impetus for development of many new achievable schemes and outer bounding techniques [11, 41, 24, 42, 15, 31, 40, 8, 19, 37, 45, 4, 7, 1]. Among these, [8] and [19] were instrumental in presenting the linear deterministic channel model used within this work, and establishing its close relationship with the Gaussian interference channel. Two desirable consequences of using the linear deterministic model will be discussed in the context of our paper:

1. Approximately-optimal layered approaches like the Han-Kobayashi code of [19] and the lattice approach of [7] are revealed naturally in the deterministic model.

2. The intimate relationship between the capacity regions of the deterministic channel and Gaussian channel and near-equivalence in the high-SNR regime (generalized degrees of freedom), provides intuition for solving real systems.

In [9], the authors introduced interference alignment (IA), a transmission scheme that is degrees of freedom optimal. Although recent IA work also considers limited knowledge conditions, e.g., the no-channel-state-information [28] and "blind" [23] cases, we consider cases without knowledge of channel statistics and our performance metrics (capacity and approximate capacity *regions*) reveal more about the performance capabilities of the network than degrees of freedom which is a single value that is not specific to a channel state.

Another way to model channel uncertainty is the approach taken in compound channels [6], which specifies a set of possible channel states and the objective to define a scheme that will maximize rate regardless of the actual channel state. Within this domain, the work of [39] is closest to our own. In their work, the authors study the compound interference channel and define an achievable scheme such that the gap between the scheme and the capacity of the Gaussian IC is bounded by a constant. However, the uncertainty set of possible channel states in the compound IC is synchronized among the nodes. In contrast, our contribution emphasizes transmitters having different uncertainty sets.[1]

[1] For one of the seven views considered in our work the results can be derived from the results of [39] because the views are identical and the responses to the system may be synchronized by

Local views in an interference channel are relatively new approach of modeling incomplete and mismatch knowledge of network state and analyzing their impact in communication networks [3, 2]. So far the emphasis has been on notions of sum-rate. We instead consider a full capacity region, thereby taking a first step towards considering alternate notions of optimal rate allocation, such as fairness metrics which require knowledge of the full feasible set of rates [33]. Additionally, the uncertainty model in [3,2,48] quantified the locality of the view by counting the number of "hops" of information. Our approach is arguably more general in the sense that we do not assume a mechanism for acquiring the view, and we instead examine a comprehensive subset of views that each results from a different knowledge acquisition mechanism.

Cooperation between nodes has been considered in a number of forms dating back to the relay channel considered in [17]. Interest in the relay channel as a cooperative network topology enjoyed a surge relatively recently [43,44,35,34,38,25,26]. Our own model for cooperative base stations may be better modeled by vector- or multiple antenna channels. In particular the Gaussian vector multiple access channel [12, 56] and Gaussian vector broadcast channels [16,10,49,50,55,54] generalize the single-input multiple-output multiple access and multiple-input single-output broadcast channels that result from base station cooperation. More recently, concrete applications of cooperation between base stations is being investigated both in algorithm and protocol design [22], and in test deployments [27]. The most thematically similar work is [51,52] where full channel knowledge but limited cooperation is assumed — in a sense our work, which analyzes limited channel knowledge and full cooperation, complements this literature nicely.

common knowledge.

1.2 Technical & Practical Merit

This dissertation is a theoretical study of a fundamental problem in wireless communications. Though seemingly idealized, our models and mathematical approach are actually driven by the desire to better understand how to improve one of the predominant commercial network architectures: cellular networks.

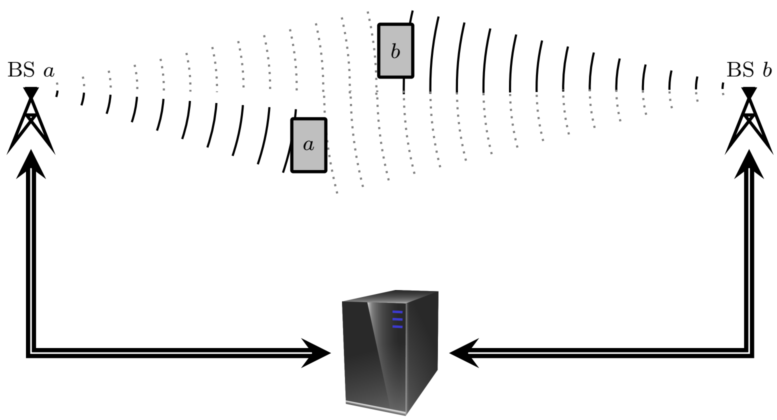


Figure 1.1: Inter-cell interference and cooperative backhaul link: In this depiction of base station cooperation for downlink communication, the cooperation may involve sharing of knowledge between base stations, as well as centralization of control. In the ideal coordinated multipoint scenario, mobiles are essentially associated with both base stations, and the interference depicted by dotted rays may be used constructively.

Cellular networks comprise an immense and lucrative world-wide industry and the cellular base-station centric network architecture will be a likely fixture in communication networks for the foreseeable future. Recently, cellular networks have evolved from carrying only voice data to a more all-purpose communication network, and now often serve as the interface between a mobile user and the broader Internet. In order to meet increased bandwidth demands, new protocol standards, such as WiMAX, LTE, and LTE-Advanced, have emerged and are attractive methods of efficiently allocating both uplink and downlink time-frequency resources *within each cell*. However, the performance gains promised for single cell design are accompanied by a caveat: unlike the 2- and 3G systems they replace, these new standards typically recommend frequency reuse factor of 1.

Since previous systems employed frequency reuse of a higher order,[1] the effective distance between cells (or sectors) operating in the same frequency band was larger than the distance between adjacent cell towers. By employing a frequency reuse factor of 1, the number of neighboring cells using the same frequency resources increases drastically and the geographic distance between these cells is reduced, increasing both the number and signal power of potential sources of inter-cell interference. This is a particularly important consideration at the boundaries of cells, where mobiles that already suffer from signal loss due to larger distance from the base station are faced with inter-cell interference signal power on the same order as the signal of interest.

With our system model, we capture only some of the most salient features of the challenges faced. This section provides real-world context and explains the motivation behind features of our approach.

1.2.1 Local Views

As alluded to in the opening of this dissertation, our contributions are derived from analysis of our local view model for node knowledge. We take this opportunity to contrast our approach to modeling node knowledge with the approaches of prior work.

Table 1.1 contrasts some of the common approaches to modeling node knowledge, specifically channel state knowledge. We note that while this thesis takes an all-or-nothing approach to knowledge of a channel gain, granular or quantized knowledge may be incorporated directly into the local view model. At its core, the most salient aspects of the local view model are non-reliance on an assumed a priori channel distribution and potential mismatch between uncertainty (views) of nodes.

We do not claim our approach is universally more appropriate than others. However, we note that incomplete and mismatched knowledge is a characteristic of the inter-cell interference problem, since cell base stations currently do not train for chan-

[1] This is especially true for GSM-based technologies. Although CDMA-based technologies allowed for frequency reuse factor of 1, this practice has only recently gained popularity.

nel states of neighboring cell users (incompleteness), and as of yet sharing of state information between cells occurs at a very coarse level (mismatch) [20].

The non-reliance on prior distribution is also an attractive characteristic of our model, especially when the underlying distribution is poorly understood, the distribution is non-ergodic, or the time-scale for evaluating performance is short. This last point in particular is relevant to the tighter deadlines of streaming and on-demand data, especially high bandwidth media like video, which is becoming an increasingly large contingent of general wireless traffic [13].

	Fast-fading	Quasi-static Fading	Compound	Local View
Types of node knowledge:				
Incomplete	Y	Y	Y	Y
"Noisy"	Y	Y	N	N
Mismatched	Y	Y	N	Y
Bayesian	Y	Y	N	N
Performance Metrics:				
Examples	Ergodic Capacity	ϵ-Outage Capacity, Diversity, DoF, DMT	Capacity, DoF	(§2.5)
Timescale	Long	Long[1]	Short	Short

Table 1.1: Common models for channel state knowledge

1.2.2 2-User Interference Channel & One-Sided Cooperation

The two-user interference channel network topology studied in this work is the smallest fundamental unit of any interference network. We work with this small scale model for two main reasons. First, the analysis of only two interfering transmit-receive pairs is considerably more straightforward than analysis of a full cellular deployment. Moreover, if the performance in a two-user interference channel is limited by the incomplete

[1] While it can be said that ε-outage capacity and diversity order are qualified performance metrics that describe a single short timescale period, in order for the empirical performance (how often in outage) to typical of the prescribed ε, a longer timescale is needed.

and mismatched local view, performance in a larger, more complex network will be definitely as limited.

Second, under the current OFDM-based intra-cell management of resources, the allocation of orthogonal time-frequency slots within one cell may lead to an unintended statistical multiplexing effect with the mobiles of a neighboring cell. This implies that while there is likely non-negligible interference for each user, the number of sources contributing to this interference is not large.

Many approaches have been proposed to address the increase in inter-cell interference resulting from more aggressive usage of frequency resources, however almost all can be considered as varying degrees of *base station cooperation*. That cooperation occurs only between base stations is consistent with our cooperative models (§2.3) and logical in the sense that base stations are large infrastructure nodes within the network, with preexisting wired connections. Some examples of proposed but as yet undeployed approaches include cross-cell scheduling of time-frequency OFDM slots, single-cell beamforming and zero-forcing, and inter-cell virtual multiuser MIMO.

Employing any of these approaches requires at least some channel knowledge, and often this knowledge is unavailable. The solution that guarantees system-wide optimality — full centralization of training, resource allocation, encoding, signaling, and decoding — is unrealistic, and thus we arrive at the impetus of the primary topics of this dissertation.

With respect to single-cell approaches, such as beamforming and zero-forcing, our analysis of local view interference channels (Chapter 3) can illustrate how much added knowledge is required to guarantee a gain in performance. For instance, whereas downlink beamforming requires only knowledge from one base station to its intended mobile user, downlink zero-forcing also requires knowledge of the link from base station to mobile of the neighboring cell. Our approach allows a network architect to interpret the benefits of additional channel training, and weigh these benefits against the associated infrastructure cost. Moreover, even approaches such as cross-cell scheduling require an understanding of what the fundamental limits of cooperation are for any pair of links scheduled in the same time-frequency slot. Knowledge of the local view capacity region provides an understanding of performance even when not all of the channels are known.

The bulk of the work considering sophisticated virtual MIMO or coordinated multi-point approaches studies full channel state knowledge and a bandwidth limited link between base stations. We contend that a more important performance bottleneck to consider is the overhead associated with learning the channel. Therefore, our study of cooperative interference mitigation (Chapter 4 and Chapter 5) provides a complementary approach to that in the existing literature. More importantly, our approach allows us to comment on what type of information is more useful to share (message or channel state) on a cooperative connection between base stations.

We intend for the results presented within this dissertation to provide wireless network architects and standards bodies with a more comprehensive understanding of the impact of increased or decreased node knowledge, allowing for improvements in system-wide efficiency in the next and future generations of wireless networks.

1.3 Contributions & Thesis Overview

The contributions of this dissertation help to answer three questions:

Question 1: Without any base station cooperation, how much knowledge do nodes need to have in order for protocols to perform better than the orthogonalized approaches, such as time-division multiplexing (TDM), already implemented in practice?

Question 2: In the uplink scenario with base station cooperation, how much knowledge do mobiles need in order to make use of the added capacity that base station cooperation provides?

Question 3: If we allow base station cooperation in downlink, how does sharing only local views help? How much does sharing only messages help? How does this compare with an idealized coordinated multipoint system?

Chapter 2 describes our mathematical model for the problem we consider. The model includes a model of the physical channel (the two-user interference channel), node knowledge (local views), and cooperative modes. Chapter 2 also provides mathematical preliminaries for analysis of capacity regions and clarifies our distributed notion of capacity with a minimum performance criterion. To further clarify the concepts used in this dissertation, we provide a toy example at the end of Chapter 2.

Chapter 3 is dedicated to answering Question 1. We derive exact local view capacity regions for a class of linear deterministic channels modeling two interfering streams. The seven local views considered can be classified into two categories: first where opportunistic coding schemes can exceed the TDM rate region and second where TDM is the optimal scheme.

In analyzing local view interference channels, we develop a number of techniques for information theoretic analysis. The techniques for defining outer bounds are presented in the beginning of Chapter 3, and may be of use in analyses of other network scenarios. More importantly, our techniques facilitate transition of results from the simpler linear deterministic model to the physically more accurate Gaussian channel models. As a result, the capacity regions for each local view linear deterministic channel also provide the basis for approximate capacity region characterizations of the local view Gaussian interference channel. We analyze the gaps between the capacity regions of the two models, which allows us to characterize the generalized degrees of freedom regions of the Gaussian IC with local views.

In Chapter 4, we address Question 2. We find considerably more opportunities for increased rates when mobiles are able to assume that base stations share their channel outputs. The only scenario where no such gain is found is the scenario where mobiles have the most limited knowledge of network state: mobiles only know the gain on the direct link to their respective base stations. Although the capacity region is enlarged by receiver cooperation in all other local views besides the most limited one, we also find that full degrees of freedom (multiplexing gain of two) cannot be guaranteed by anything less than a full view, i.e., when both mobiles know the full channel state.

Chapter 5 describes our steps towards answering Question 3. The downlink scenario offers more possible modes of cooperation: Should we share knowledge about the network state or the messages themselves? For some cases of local views and cooperative modes, we are able to apply previous results directly. For others, we find outer bounds on the capacity region for all views considered, which allow us to comment on the most one might hope for, assuming a specific type of base station cooperation. For some scenarios we are also able to provide approximately capacity achieving schemes, which are optimal in a generalized degrees of freedom sense. One interesting result is the approximate reciprocity that exists between our capacity outer bounds for message-only transmitter cooperation and the capacity region of local view interference channels with receiver cooperation.

The content of Chapters 3–5 are necessarily technical, therefore in order to provide the reader with a clearer picture of the intuition gained from our findings, we end each of these chapters with a series of remarks explaining the impact of the results and their relation to the mitigation of inter-cell interference.

Chapter 6 summarizes and discusses future directions, as well as possible applications of results. The bulk of mathematical rigor is relegated to proofs in Appendix A. Mathematics presented in body of chapter either illustrates a point or describes a technique that is of technical interest in its own right.

Chapter 2

General Preliminaries

2.1 Channel Model: 2-User Interference Channel

As the most fundamental unit of any wireless network with interference, the focus of this thesis is the two-user interference channel. We consider both Gaussian and linear deterministic interference models. Our focus is on the Gaussian two-user interference channel, as it is the canonical model of the wireless networks. However, the linear deterministic model of [5, 8], whose structure and performance approximates that of the Gaussian channel, is useful in providing clarity in the development of our results. Both models are described here.

2.1.1 Gaussian Interference Channel

The default model for describing interfering transmissions in a wireless medium is the two-user Gaussian interference channel (GIC) shown in Figure 2.1. The GIC consists of two transmitter-receiver pairs, labeled a and b, and four point-to-point *links* — two direct and two interfering. The signal strengths of each point-to-point link are represented by the four complex gain values h_{aa}, h_{ab}, h_{ba}, and h_{bb}, collectively referred to as H. The Gaussian model succinctly captures three main features of wireless transmission:

- Broadcast — signals traverse free space and reach both intended and unintended receivers albeit with differing levels of attenuation.
- Superposition — destination nodes receive the sum of electromagnetic fluctuations of multiple signals.
- Noise — thermal noise within the receiver circuit reduces the fidelity of the received signal.

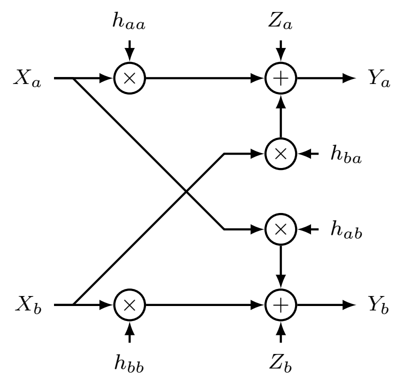


Figure 2.1: Gaussian Interference Channel

The relationship between complex channel inputs X_a and X_b and the received channel outputs Y_a and Y_b is given by

$$Y_a = h_{aa}X_a + h_{ba}X_b + Z_a, \tag{2.1}$$

$$Y_b = h_{bb}X_b + h_{ab}X_a + Z_b, \tag{2.2}$$

where we have a length-n codeword power constraint $\frac{1}{n}\sum_{t=1}^{n} \|X_i[t]\|^2 \leq 1$ for $i \in \{a, b\}$, and Z_a and Z_b are zero-mean, unit-variance Gaussian random variables.

2.1.2 Linear Deterministic Interference Channel

The linear deterministic model captures the broadcast and superposition aspects of the Gaussian model, while abstracting the receiver noise into a signal level "floor" at each receiver. In doing so, the effects of noise become a constant effect, isolating the impact of interference on reliable communication.

A linear deterministic interference channel (LDIC) is defined by four integer point-to-point link gains $(g_{aa}, g_{ab}, g_{ba}, g_{bb})$, each related to a GIC gain as $g_{ij} = \lfloor \log(\|h_{ij}\|^2) \rfloor^+$. The deterministic gain essentially quantifies the signal-to-noise ratio (SNR) of each link on a decibel scale, however this value may be interpreted in other ways:

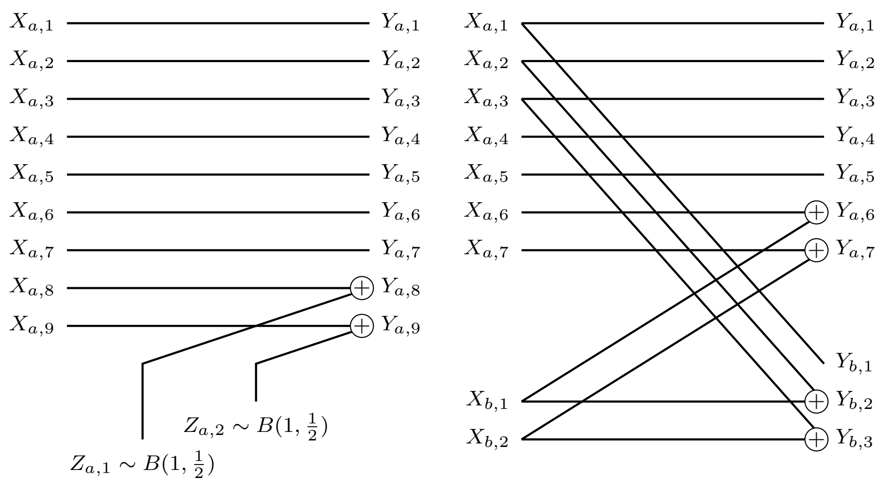
- an approximation of the number of binary values that can be reliably conveyed in the respective point-to-point GIC link,
- a quantification of how many bits of significance can be discerned from a noisy signal.

The first interpretation is easily demonstrated by comparing the LDIC gain to the point-to-point capacity of the Gaussian link [46]:

$$g_{ij} \triangleq \lfloor \log\left(|h_{ij}|^2\right) \rfloor^+ \approx \log\left(|h_{ij}|^2\right) \approx \log\left(1 + |h_{ij}|^2\right), \tag{2.3}$$

where the second relationship holds for sufficiently large $\|h_{ij}\|$. The second interpretation is graphically depicted in Figure 2.2(a).

For the LDIC, channel inputs and outputs thus take the form of binary tuples; e.g., $(X_{a,1}, X_{a,2}, \ldots)$, is the channel input of Transmitter a where $X_{a,1}$ is the most significant bit. Per channel use, each transmitter transmits a q-tuple of binary values, $(X_{i,1}, X_{i,2}, \ldots, X_{i,q})$, and if only Transmitter i transmits, then Receiver j receives the g_{ij} most significant entries of Transmitter i's transmission. When both transmitters transmit simultaneously, the two vectors of potentially differing length seen at Receiver j are aligned such that the least significant bits ($X_{a,g_{aj}}$ and $X_{b,g_{bj}}$) of the two transmissions coincide, and as in [8], coincidental received bits are combined via modulo-2 addition. Figure 2.2(b) depicts an example of a LDIC.



(a) Point-to-point Linear Deterministic Model

(b) LDIC

Figure 2.2: Linear Deterministic Interference Channel: (a) Single-user linear deterministic channel with inclusion of signal levels below the “noise floor”, (b) An example of a linear deterministic interference channel

When considering the deterministic channel we refer to single-channel-use input (output) tuples of a deterministic Transmitter i (Receiver j) collectively as $\mathbf{X}_i$ ($\mathbf{Y}_j$), and to length-n vectors of channel inputs (outputs) — e.g., when considering n channel uses — as $\mathbf{X}_i^n$ ($\mathbf{Y}_j^n$) respectively. We also refer to the collection of all four channel gains as the *channel state* $G = (g_{aa}, g_{ab}, g_{ba}, g_{bb})$, and express mathematically the input-output relationship of the deterministic channel in the form of shift matrix operations and element-wise modulo-2 addition. Notice the bold weight of the channel input and output values, and also use of G instead of H to distinguish the linear deterministic variables from those of the Gaussian model.

$$\mathbf{Y}_a = S_{q_a}^{q_a - g_{aa}} \mathbf{X}_a \oplus S_{q_a}^{q_a - g_{ba}} \mathbf{X}_b, \tag{2.4}$$

$$\mathbf{Y}_b = S_{q_b}^{q_b - g_{bb}} \mathbf{X}_b \oplus S_{q_b}^{q_b - g_{ab}} \mathbf{X}_a, \tag{2.5}$$

where $q_a = \max(g_{aa}, g_{ba})$, $q_b = \max(g_{bb}, g_{ab})$, and S_q is the $q \times q$ shift matrix

$$S_q = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix}.$$

In the interest of brevity, we occasionally make use the shorthand

$$\mathbf{Y}_j = \mathbf{X}_a \oplus \mathbf{X}_b, \tag{2.6}$$

to denote the input-output relationship of the channel when the channel structure is otherwise clear.

2.2 Knowledge Model: Local Views

Under our local view model, Transmitter a's knowledge is composed of only a subset of all the gains in the network. When a link gain is known, it is known perfectly (without error), and when it is unknown, Transmitter a has no knowledge of its value besides its support, which is the complex field for the Gaussian channel model and all non-negative integers for the deterministic channel model. We call the subset of channel gains known to Transmitter a its *local view*, and denote it for the Gaussian channel model as $\widehat{H}_a$. When specifying the many possible views, the symbol $\varnothing$ is used for unknown gains. As an example, if Transmitter a knew all of the gains besides that of the direct link between Transmitter b and Receiver b, we would denote the relationship as

$$\widehat{H}_a = (h_{aa}, h_{ab}, h_{ba}, \varnothing).$$

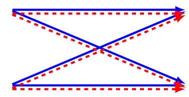
The same conventions hold for Transmitter b, as well as for the deterministic channel model except we denote channels and views with G instead of H to distinguish between gains of the two different models.

Note that the view $\widehat{H}_a$ describes a subspace of the full parameter space of possible channel states. Since we have assumed error-free knowledge, the actual channel state H lies within the subspace described by each transmitter's local view. Moreover, the intersection of the subspaces described by the views of both users specifies the network-wide view of the channel state if transmitters could collaborate, and therefore must be non-empty since the actual channel state lies within this intersection.

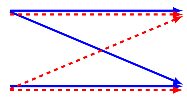
We restrict our study to cases where structures of the two views are *symmetric*: e.g., if gain h_{ij} is (un)known to Transmitter a, then h_{ji} is (un)known to Transmitter b. Symmetric views are only a subset of the full set of cases that may be considered. However, for the two-user IC, symmetric views describe cases where the method used by each transmitter to learn about the network is the same. All eight symmetric views are depicted in Figure 2.3.

The receivers are assumed to have sufficient knowledge to accommodate transmitter decisions and decode messages coherently. It is important to clarify that both transmitters are aware of the size of the network (two links in isolation), as well as the structure of the view of the other transmitter (which link gains are known and unknown).

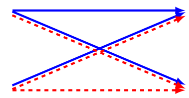
We emphasize that our objective is not the development of a training or network learning mechanism. Instead, our approach is to determine the fundamental limits of communication, assuming that the view available to each node *results from* a particular network measurement architecture. For example, a network where transmitters rely on passive measurements taken at the transmitter results in the view depicted in Figure 2.3(e), whereas a network relying on measurements taken at the receiver (and relayed back to the transmitter) might result in Figure 2.3(g). Therefore, each view



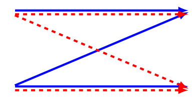
(a) View 0 — Full CSI



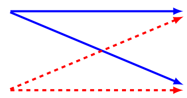
(b) View 1



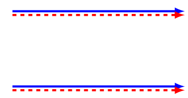
(c) View 2



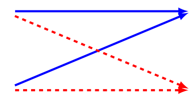
(d) View 3



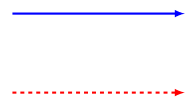
(e) View 4



(f) View 5



(g) View 6



(h) View 7

Figure 2.3: Local View Diagrams: Solid blue link gains are known by Transmitter a and dashed red are known by Transmitter b.

we consider represents the per-node knowledge resulting from a potential network architecture.

2.3 One-Sided Cooperation Model

In Chapters 4 and 5 we study how base station cooperation can alleviate some of the challenges posed by insufficient local view. Two cooperation models are studied, each associated with a mode of cellular communication network.

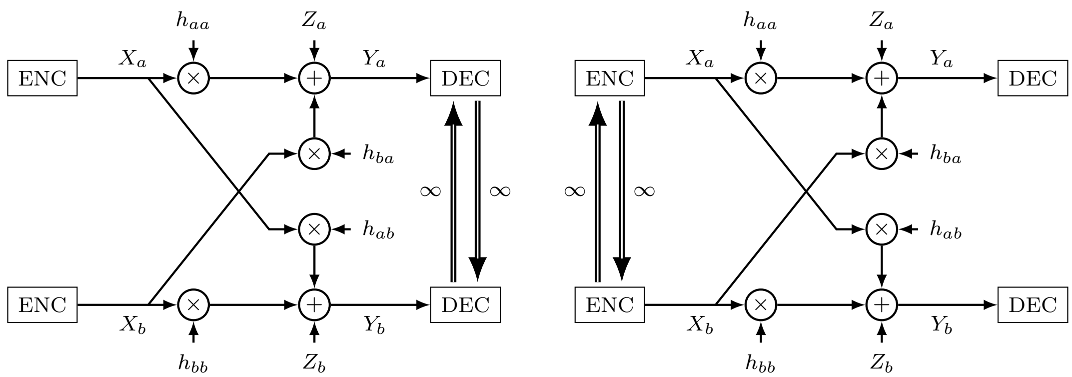
Uplink — Receiver Cooperation (Figure 2.4(a)): Our receiver cooperation model adds an infinite-rate noiseless link between the receivers of the interference channel. Channel outputs are shared on this link, and consequently, the local view IC may be reinterpreted as a mathematically equivalent 1×2 single-input multiple-output (SIMO) multiple-access channel with local views.

Downlink — Transmitter Cooperation (Figure 2.4(b)): Our transmitter cooperation model adds an infinite-rate noiseless link between transmitters. Although we assume unconstrained bandwidth, we do examine constraints on the *type* of information that is shared. The first case we consider is the sharing of local views or channel state knowledge. This approach results in local views at each transmitter that are potentially complete and, more importantly, matched. Matched local views at each transmitter allow protocols to exercise a coordinated response, which we show improves performance.

The second type of knowledge that we consider is message data. In this scenario, transmitters must still determine their channel inputs independently based solely upon their (mismatched) local views, but may use message data to accommodate interference, or assist the other stream. In essence, the message-only transmitter cooperation IC model can be viewed as a specific type of relay network [17] with local views, where, in addition to the known link gains depicted in Figure 2.3, two infinite-rate directional links between transmitters are always known (to exist and to have infinite bandwidth) to both transmitters (Figure 2.5).

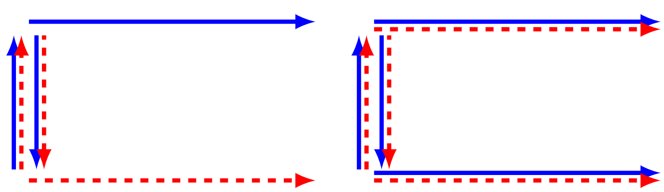
Finally, we consider the case where both local views and message data may be shared on the cooperative link. This results in either a MISO broadcast (the capacity of which is known [54]) or a special case of compound MISO broadcast channels.

Though in practice an infinite-rate link may seem unrealistic, by choosing these models we seek to isolate the impact of local views on cooperation in interference scenarios. Furthermore, in many networks with infrastructure (e.g., cellular networks), sharing of message information may be significantly less costly than the act of learning channel information. For the specific case of cellular networks, one may consider our transmitter cooperation and receiver cooperation models as two modes of base station cooperation for downlink and uplink, respectively.



(a) Receiver Cooperation (b) Transmitter Cooperation

Figure 2.4: One-Sided Cooperation Models: (a) Receiver cooperation modeling base station cooperation in uplink, (b) Transmitter cooperation modeling base station cooperation in downlink



(a) (b)

Figure 2.5: Local Views with TX cooperation: (a) View 7 with message-only cooperative links, (b) View 5 or View 7 allowing sharing of channel knowledge.

2.4 Mathematical Preliminaries

Each transmitter uses an encoding function, $c_{i,n}$, to encode a message, m_i, drawn independently from the set $M_i = \{1, \ldots, 2^{nR_i}\}$ into a codeword of n symbols, $\mathbf{X}_i^n = (\mathbf{X}_i[1], \ldots, \mathbf{X}_i[n])$, subject to a unit power constraint $\frac{1}{n}\sum_{t=1}^{n} \|X_i[t]\|^2 \leq 1$ for the Gaussian model.

Each receiver observes its channel outputs, $(\mathbf{Y}_i[1], \ldots, \mathbf{Y}_i[n])$, and uses a decoding function $f_{i,n}$ to arrive at an estimate, $\widehat{m}_i \in M_i$, of the transmitted message, m_i. An error occurs whenever $\widehat{m}_i \neq m_i$. The average probability of error for User i is given by

$$\epsilon_{i,n} = E[\Pr(\widehat{m}_i \neq m_i)],$$

where the expectation is taken with respect to the random choice of the transmitted messages m_a and m_b.

A rate pair (r_a, r_b) is achievable if there exists a family of pairs of codebooks $\{c_{a,n}, c_{b,n}\}_{n\in\mathbb{N}}$ indexed by the block length n, with codewords satisfying input constraints, and decoding functions $\{f_{a,n}(\cdot), f_{b,n}(\cdot)\}_{n\in\mathbb{N}}$, such that the average decoding error probabilities $\epsilon_{a,n}, \epsilon_{b,n}$ vanish as block length n goes to infinity. By applying Shannon's coding theorem for the point-to-point channel, the set of achievable rate points can be determined by the following.

Lemma 1. *[8, Lemma 1] The rate point (r_a, r_b) is achievable if and only if for every $\epsilon > 0$ there exists a block length n and distributions $p(\mathbf{X}_a^n)$ and $p(\mathbf{X}_b^n)$ such that*

$$nr_a - \epsilon \leq I(\mathbf{X}_a^n; \mathbf{Y}_a^n), \tag{2.7}$$

$$nr_b - \epsilon \leq I(\mathbf{X}_b^n; \mathbf{Y}_b^n). \tag{2.8}$$

The capacity region $\mathcal{C}$ of the interference channel is the closure of the set of all achievable rate pairs.

2.4.1 Generalized Degrees of Freedom

Within this thesis we also make use of the performance metric called Generalized Degrees of Freedom (GDoF). Consider a Gaussian interference channel and let the parameter $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ be defined as

$$\alpha_1 \triangleq \frac{\log(\|h_{bb}\|^2)}{\log(\|h_{aa}\|^2)}, \tag{2.9}$$

$$\alpha_2 \triangleq \frac{\log(\|h_{ba}\|^2)}{\log(\|h_{aa}\|^2)}, \tag{2.10}$$

$$\alpha_3 \triangleq \frac{\log(\|h_{ab}\|^2)}{\log(\|h_{aa}\|^2)}. \tag{2.11}$$

and $\mathcal{C}(h_{aa}, h_{ab}, h_{ba}, h_{bb})$ be the capacity region of the complex Gaussian IC given by gains $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$.

The GDoF region was defined in [19] as

$$\mathcal{D}(\alpha) = \lim_{\substack{H\to\infty \\ \alpha \text{ fixed}}} \left\{ \left(\frac{r_a}{\log\left(\|h_{aa}\|^2\right)}, \frac{r_b}{\log\left(\|h_{bb}\|^2\right)} \right) : (r_a, r_b) \in \mathcal{C}\left(h_{aa}, h_{ab}, h_{ba}, h_{bb}\right) \right\} \quad (2.12)$$

The expression (2.12) essentially defines a region approximating the set of achievable pre-log factors for channels with similar interference behavior. For example, if the pair $\left(\frac{1}{2}, \frac{3}{4}\right)$ is in the GDoF region, then that means that it is possible for User a to achieve approximately $\frac{1}{2}$ and User b to achieve approximately $\frac{3}{4}$ of their non-interfered user rate. In the case were interference can be mitigated completely, the GDoF point $(1, 1)$ is achievable. In the case where interference can not only be mitigated, but also used to one's benefit (i.e., transmitter cooperation) it is possible to achieve a GDoF point even greater than $(1, 1)$.

We note that throughout the dissertation, any GDoF results are computed directly from the more precise capacity characterizations. Because the computations are straightforward and rely only on identifying dominant terms within the log, and normalizing by the appropriate value, we omit explanations of these computations.

2.5 A Distributed Notion of Capacity

2.5.1 Distributed Policies

In a centralized network, encoding functions may be designed jointly to match the channel state. On the other hand, in our model each transmitter selects its encoding function based on its respective local view. Specifically, the encoding function used, $c_{i,n}(m_i; \widehat{G}_i)$, is a function of the local view, which implies that the resulting rates, $r_a(\widehat{G}_a)$ and $r_b(\widehat{G}_b)$, and channel input distributions, $p(\mathbf{X}_a; \widehat{G}_a)$ and $p(\mathbf{X}_b; \widehat{G}_b)$, are also dependent on local view.

We assume that the mapping from view, $\widehat{G}_i$, to encoding function, $c_{i,n}(m_i; \widehat{G}_i)$, is both *deterministic* and *globally known*; i.e., although Transmitter a may not know Transmitter b's exact choice of codebook and rate (due to mismatched views of a and b), Transmitter a knows how b would respond to a particular channel state.

The deterministic mapping from view to codebook is analogous to a predetermined protocol or *policy* either agreed upon by the two users, or specified by the network architecture. Therefore, throughout the paper we refer collectively to the mappings $c_{i,n}(m_i; \widehat{G}_i)$, $r_i(\widehat{G}_i)$, and $p(\mathbf{X}_i; \widehat{G}_i)$ as the policy of Transmitter i for $i \in \{a, b\}$.

The concept of a policy couples the uncertainty of the interferers' choice of encoding function to the uncertainty in channel state. Accordingly, the ability to coordinate encoding functions (and the resulting performance) remains dependent on each transmitter's local view. Under our formulation, we define achievability of a pair of policy-defined rates, $r_a(\widehat{G}_a)$ and $r_b(\widehat{G}_b)$, by extending Lemma 1 to apply to view-dependent encoding functions and requiring achievability of the respective encoding functions for all channel states consistent with the local view considered. Mathematically, achievability requires the existence of view-dependent input distributions $p(\mathbf{X}_a; \widehat{G}_a)$ and $p(\mathbf{X}_a; \widehat{G}_a)$, such that for all G

$$nr_a(\widehat{G}_a) - \epsilon \leq I(\mathbf{X}_a^n; \mathbf{Y}_a^n), \tag{2.13}$$

$$nr_b(\widehat{G}_b) - \epsilon \leq I(\mathbf{X}_b^n; \mathbf{Y}_b^n). \tag{2.14}$$

2.5.2 Minimum Performance Constraint

The inequalities (2.13) and (2.14) are necessary conditions for achievability of the target rates and existence of encoding functions dictated by a pair of policies. To these two conditions we add a third criterion:

Minimum Performance Criterion: Let $\overline{\mathcal{R}}^{\mathsf{TDM}}$ be defined as the Pareto optimal frontier (non-zero rate boundary) of the regions achieved using time-division multiplexing (TDM) for the channel considered;[1] these regions are explicitly defined in §3.2 for the non-cooperative case, and §4.1 and §5.1 for the two cooperative cases. For each policy considered, there exists some $(r_a^{\mathsf{TDM}}, r_b^{\mathsf{TDM}}) \in \overline{\mathcal{R}}^{\mathsf{TDM}}$ such that for every channel state H, and views $\widehat{H}_a$ and $\widehat{H}_b$,[2]

$$r_a(\widehat{H}_a) \geq r_a^{\mathsf{TDM}}, \tag{2.15}$$

$$r_b(\widehat{H}_b) \geq r_b^{\mathsf{TDM}}. \tag{2.16}$$

The TDM minimum performance criterion provides context for the concept of a capacity region. For all the local views we consider (ref. Figure 2.3), TDM is a viable policy and a point of reference for the performance of any other policy. Moreover, if a local view is shown to have a strictly larger capacity region under the minimum performance criterion, then there exists a policy which universally outperforms TDM or any other orthogonalized scheme, across all channel realizations.

Thus, without the TDM minimum performance criterion, the concept of a capacity region for channel state G can be misleading. In fact, a policy *always* exists that achieves any rate point in the full view capacity region for a given network state. However, as we demonstrate in the next section, use of such a policy often comes at the cost of what can be achieved in another channel state.

For a geometric visualization of the minimum performance constraint, consider a $2s$-dimensional region describing the rates achievable for s channel state realizations — the two dimensions per channel state are for the rates achieved by two users in each state. With a full view, each transmitter's policy is able to adjust to every

[1]In this dissertation we use TDM as a concrete example of orthogonalization-based schemes. However, our results still hold if we assume any other orthogonalization-based approach as our baseline policy.

[2]Notationally, we have assumed a Gaussian interference model, however this criterion is enforced for all channels studied.

specific channel state, and therefore the question of achievability of a set of policies is separable between the different channel states. However, with a local view policy each transmitter's action may be the fixed for a subset of channel states; a local view may not provide enough knowledge to distinguish different states. If both transmitters experience this ambiguity, and more importantly if the unknown quantities for each user are mismatched, then the policies used are coupled across channel states, and the form of the $2s$-dimensional region is not separable.

Our two-dimensional capacity region under the TDM minimum performance constraint only partially describes the $2s$-dimensional region: it is a projection of only a portion of the $2s$-dimensional region onto the subspace corresponding to the channel state of interest. The portion projected however is not arbitrary, as it includes only $2s$-dimensional rate-tuples that achieve performance at least as good as TDM in all subspaces.

2.5.3 Example: A Local View Multiple-Access Channel

Consider the two-user linear deterministic local view multiple-access channel (LV-MAC) where users' link gains can take the value 1 or 2. The full-view capacity region for any particular channel state is

$$r_a \leq g_a, \tag{2.17}$$

$$r_b \leq g_b, \tag{2.18}$$

$$r_a + r_b \leq \max(g_a, g_b). \tag{2.19}$$

The local views are such that each user only knows its direct link gain, i.e.,

$$\widehat{G}_a = (g_a, \varnothing), \tag{2.20}$$

$$\widehat{G}_b = (\varnothing, g_b). \tag{2.21}$$

As in our LV-IC, transmitters must determine a codebook and rate given unique, incomplete views of the channel. Therefore, consider a policy that when $g_a = 2$ and $g_b = 1$ achieves the rate point $(1, 1)$, which is a corner point on the full knowledge capacity region, i.e.,

$$r_a(\widehat{G}_a)|_{g_a=2} = 1, \tag{2.22}$$

$$r_b(\widehat{G}_b)|_{g_b=1} = 1. \tag{2.23}$$

In order to satisfy (2.17)–(2.19) for the events $G = (1, 1)$ and $G = (2, 2)$,

$$r_a(\widehat{G}_a)|_{g_a=1} = 0, \tag{2.24}$$

$$r_b(\widehat{G}_b)|_{g_b=2} \leq 1. \tag{2.25}$$

For the channel states $G = (1, 1)$, $G = (2, 1)$, and $G = (2, 2)$, the policy results in rate points on the boundary of the respective ideal capacity regions. However, for the case where $G = (1, 2)$, the resulting rate point, $r = (0, 1)$, is not only an interior point, but also less efficient than TDM. Therefore, although the policy designed thus far outperforms TDM for some channel states, the performance gain comes at the expense of what may occur in other states. Does a policy exist where rate points outside the TDM can be achieved at no cost? For the LV-MAC with views given by (2.20) and (2.21), the answer is no.

Consider the two channel states $G = (1, 1)$ and $G = (2, 2)$. In each, TDM is capacity achieving even with a full view. Let the time-division parameters in state $G = (1, 1)$ be defined as $\tau_a(1)$ and $\tau_b(1)$, where $\tau_a(1) + \tau_b(1) = 1$. Similarly, we define $\tau_a(2)$ and $\tau_b(2)$, where $\tau_a(2) + \tau_b(2) = 1$. The rates resulting from this policy are $r_a(\widehat{G}_a)|_{g_a=s} = s\tau_a(s)$ and $r_b(\widehat{G}_b)|_{g_b=t} = t\tau_b(t)$.

Assume that $\tau_a(1) \leq \tau_a(2)$, which implies $\tau_b(2) \leq \tau_b(1)$. We now consider the channel state $G = (1, 2)$ and notice

$$\frac{r_a(\widehat{G}_a)}{g_a} + \frac{r_b(\widehat{G}_b)}{g_b} = \tau_a(1) + \tau_b(2) \leq 1, \tag{2.26}$$

with equality if and only if $\tau_a(1) = \tau_a(2)$ and $\tau_b(2) = \tau_b(1)$, i.e., if the rate achieved in each channel state is at least as good as TDM, then not only are the capacity regions of all four possible channel states the TDM region, but also all four states are tied to the same operating point (time-division allocation).

In fact, the confinement to a single time-division regardless of channel state holds true for a more general case as well: the capacity of the K-user deterministic multiple-access channel where each transmitter only knows the gain of its direct link cannot universally exceed the region achieved by TDM. Moreover, all channel states are bound to the same time divisions. The proof can be found in Appendix A.1.

Theorem 2 (LV-MAC Capacity Region)**.** *Let a LV-MAC be defined as a K-user multiple-access channel where for each transmitter, $\widehat{G}_k = (\varnothing, \ldots, \varnothing, g_k, \varnothing, \ldots)$. If*

$$\sum_{k=1}^{K} \frac{r_k(\widehat{G}_k)}{g_k} \geq 1 \ \forall \ G, \tag{2.27}$$

then

$$r_k(\widehat{G}_k) = g_k \tau_k \tag{2.28}$$

for a set of τ_k satisfying $\sum_{k=1}^{K} \tau_k = 1$. Conversely, if the policies are such that there exists G where

$$\sum_{k}^{K} \frac{r_k(\widehat{G}_k)}{g_k} > 1, \tag{2.29}$$

then there also exists a G' such that

$$\sum_{k=1}^{K} \frac{r_k(\widehat{G}'_k)}{g'_k} < 1. \tag{2.30}$$

Chapter 3

Local View Interference Channels

In this chapter, we present approximate characterizations of the local view Gaussian interference channel (LV-GIC) capacity region, and the resulting generalized degrees of freedom (GDoF) regions. The chapter is organized linearly, with each section building upon the tools and results presented before.

First, we describe the techniques used to establish inner and outer bounds for the capacity regions. Our inner bounds are based on simple notions within the literature, but the key point is that policies opportunistically determine which scheme to apply, based on knowledge from the local view.

Our outer bounds are inspired by first analyzing the local view linear deterministic interference channel (LV-LDIC), and developing an appropriate methodology for constructing bounds that apply to the Gaussian model. Our bounds have two key facets: (1) "unwrapping" of the interference channel into a virtual Z-channel in order to account for uncertainty as well as the coupling between policy responses that results from limited local view, and (2) Gaussian genie signals analogous to the signal levels of the linear deterministic channel, in order to translate the intuition of the LV-LDIC into the LV-GIC domain. To our knowledge both of these are new techniques and will likely prove useful in further study of local view networks and interference channels.

We study the LV-LDIC, where the simple model isolates the effect of interference and reveals the importance of being able to align one's signal at both receivers in order to apply more sophisticated encoding mechanisms. Tight characterizations of our distributed notion of capacity region are given for all local views in Figure 2.3.

Finally, by applying a genie that captures the importance of aligning signal levels, we bound the gaps between an simple opportunistic Han-Kobayashi coding policy and the boundary of the capacity region for each view. These gaps are sometimes channel dependent, however are invariant to the strength of signaling; i.e. if all channel gains are held constant relative to each other on a decibel scale, the gap does not change with SNR. From this we find the GDoF regions for each LV-GIC.

3.1 Bounding Techniques

One approach to characterizing a capacity region is through analysis of inner and outer bounds. Though the analysis for each local view varies, the basic techniques employed are summarized in this section.

3.1.1 Inner Bounds

In this work, we reference only two types of achievable schemes: Time-Division Multiplexing (TDM) and the simple Han-Kobayashi scheme (HK) of [19].

3.1.1.1 Time-Division Multiplexing

For TDM, the transmissions of the two users are orthogonalized in time. Let τ_a and τ_b where $\tau_a + \tau_b \leq 1$ be the portions of time allotted to Transmitter a and Transmitter b respectively. If we assume that each transmitter maximizes its rate in its allotted time to transmit, each (τ_a, τ_b) defines a unique rate point. By considering all rate points, under the Gaussian IC the TDM achievable region $\mathcal{R}_G^{\mathsf{TDM}}$ is the closure of the union over $\tau \in (0,1)$ of all rate pairs (r_a, r_b) satisfying

$$r_a \leq \tau \log\left(1 + \frac{\|h_{aa}\|^2}{\tau}\right), \tag{3.1}$$

$$r_b \leq (1-\tau) \log\left(1 + \frac{\|h_{bb}\|^2}{1-\tau}\right). \tag{3.2}$$

The deterministic IC yields a similar region, albeit without the gain from efficient power management: $\mathcal{R}_D^{\mathsf{TDM}}$ is the closure of the union over $\tau \in (0,1)$ of all rate pairs (r_a, r_b) satisfying

$$r_a \leq \tau g_{aa}, \tag{3.3}$$

$$r_b \leq (1-\tau) g_{bb}, \tag{3.4}$$

which can be more succinctly written as $\mathcal{R}_D^{\mathsf{TDM}} = \{(r_a, r_b) : \frac{r_a}{g_{aa}} + \frac{r_b}{g_{bb}} \leq 1\}$.

Neither region is dependent on the interference link gains, and by construction of the linear deterministic channel from a Gaussian channel, the gap between the two regions can be shown to be at most two bits per user. Consider for User a

$$\Delta_a^{\mathsf{TDM}} = \tau_a \log\left(1 + \frac{\|h_{aa}\|^2}{\tau_a}\right) - \tau_a g_{aa} \tag{3.5}$$

$$= \tau_a \left[\log\left(1 + \frac{\|h_{aa}\|^2}{\tau_a}\right) - \lfloor \log\left(\|h_{aa}\|^2\right) \rfloor^+\right]. \tag{3.6}$$

When $\|h_{aa}\| \leq 1$, we have

$$\Delta_a^{\mathsf{TDM}} \leq \tau_a \left[\log\left(1 + \frac{\|h_{aa}\|^2}{\tau_a}\right)\right] \tag{3.7}$$

$$\leq \log(2) = 1. \tag{3.8}$$

When $\|h_{aa}\| \geq 1$, we have

$$\Delta_a^{\mathsf{TDM}} \leq \tau_a \left[\log\left(1 + \frac{\|h_{aa}\|^2}{\tau_a}\right) - \lfloor \log\left(\|h_{aa}\|^2\right) \rfloor\right] \tag{3.9}$$

$$= \tau_a \left[\log\left(1 + \frac{1}{\tau_a}\right) + 1\right] \tag{3.10}$$

$$\leq 1 + \tau_a \leq 2. \tag{3.11}$$

3.1.1.2 Simple Han-Kobayashi Codes

In general HK schemes, each transmitter splits the contents of its message into a common message and a private message. The simple HK codes of [19] use random Gaussian codebooks for both the common and private encoding functions, with a division in power between the two chosen such that the private component of the message is received at the unintended receiver "in the noise floor"; i.e., the private and common codebooks are drawn from independent zero-mean Gaussian distributions with variances $P_{i,p} = \min\left(\frac{1}{\|h_{ij}\|^2}, 1\right)$ $(i \neq j)$ and $P_{i,c} = 1 - P_{i,p}$ respectively.

At the receiver the private message of the undesired signal is treated as noise, thereby at most doubling the power of the interference-noise floor in the Gaussian case. The receiver jointly decodes both common messages and the desired private message, forming a virtual three user multiple-access channel. The resulting rate region $\mathcal{R}_G^{\mathsf{HK}}$ is approximately capacity achieving, and is given by all rate pairs (r_a, r_b)

such that $r_a = r_{a,p} + r_{a,c}$ and $r_b = r_{b,p} + r_{b,c}$ satisfying

$$r_{a,p} \geq 0 \tag{3.12}$$

$$r_{a,c} \geq 0 \tag{3.13}$$

$$r_{b,p} \geq 0 \tag{3.14}$$

$$r_{b,c} \geq 0 \tag{3.15}$$

$$r_{a,p} \leq \log\left(1 + \frac{\|h_{aa}\|^2 P_{a,p}}{1 + \|h_{ba}\|^2 P_{b,p}}\right) \tag{3.16}$$

$$r_{a,c} \leq \log\left(1 + \frac{\|h_{aa}\|^2 P_{a,c}}{1 + \|h_{ba}\|^2 P_{b,p}}\right) \tag{3.17}$$

$$r_{b,c} \leq \log\left(1 + \frac{\|h_{ba}\|^2 P_{b,c}}{1 + \|h_{ba}\|^2 P_{b,p}}\right) \tag{3.18}$$

$$r_{a,p} + r_{a,c} \leq \log\left(1 + \frac{\|h_{aa}\|^2}{1 + \|h_{ba}\|^2 P_{b,p}}\right) \tag{3.19}$$

$$r_{a,p} + r_{b,c} \leq \log\left(1 + \frac{\|h_{aa}\|^2 P_{a,p} + \|h_{ba}\|^2 P_{b,c}}{1 + \|h_{ba}\|^2 P_{b,p}}\right) \tag{3.20}$$

$$r_{a,c} + r_{b,c} \leq \log\left(1 + \frac{\|h_{aa}\|^2 P_{a,c} + \|h_{ba}\|^2 P_{b,c}}{1 + \|h_{ba}\|^2 P_{b,p}}\right) \tag{3.21}$$

$$r_{a,p} + r_{a,c} + r_{b,c} \leq \log\left(1 + \frac{\|h_{aa}\|^2 + \|h_{ba}\|^2 P_{b,c}}{1 + \|h_{ba}\|^2 P_{b,p}}\right) \tag{3.22}$$

$$r_{b,p} \leq \log\left(1 + \frac{\|h_{bb}\|^2 P_{b,p}}{1 + \|h_{ab}\|^2 P_{a,p}}\right) \tag{3.23}$$

$$r_{b,c} \leq \log\left(1 + \frac{\|h_{bb}\|^2 P_{b,c}}{1 + \|h_{ab}\|^2 P_{a,p}}\right) \tag{3.24}$$

$$r_{a,c} \leq \log\left(1 + \frac{\|h_{ab}\|^2 P_{a,c}}{1 + \|h_{ab}\|^2 P_{a,p}}\right) \tag{3.25}$$

$$r_{b,p} + r_{b,c} \leq \log\left(1 + \frac{\|h_{bb}\|^2}{1 + \|h_{ab}\|^2 P_{a,p}}\right) \tag{3.26}$$

$$r_{b,p} + r_{a,c} \leq \log\left(1 + \frac{\|h_{bb}\|^2 P_{b,p} + \|h_{ab}\|^2 P_{a,c}}{1 + \|h_{ab}\|^2 P_{a,p}}\right) \tag{3.27}$$

$$r_{b,c} + r_{a,c} \leq \log\left(1 + \frac{\|h_{bb}\|^2 P_{b,c} + \|h_{ab}\|^2 P_{a,c}}{1 + \|h_{ab}\|^2 P_{a,p}}\right) \tag{3.28}$$

$$r_{b,p} + r_{b,c} + r_{a,c} \leq \log\left(1 + \frac{\|h_{bb}\|^2 + \|h_{ab}\|^2 P_{a,c}}{1 + \|h_{ab}\|^2 P_{a,p}}\right). \tag{3.29}$$

The analogous approach in the deterministic IC is to similarly split each user's message into common and private parts, where the private message is carried by a set of the least significant set of bits, specifically all bits that are not seen at the unintended receiver (shown in Figure 3.1). As in the Gaussian IC case, receivers

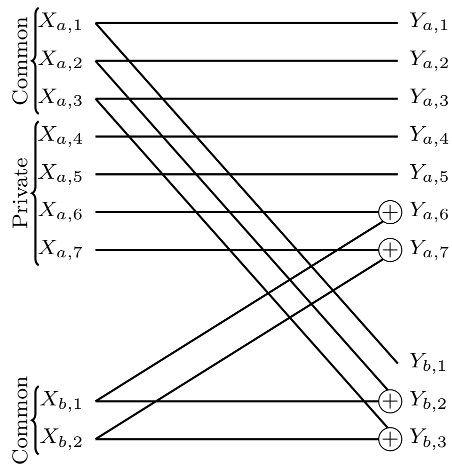


Figure 3.1: HK Coding in a Linear Deterministic IC: Note the separation of usable linear deterministic channel layers into common and private components.

decode the two common messages and desired private message as a virtual three-user MAC, resulting in the capacity-achieving rate region, $\mathcal{R}_D^{\mathsf{HK}}$, given by all rate pairs

(r_a, r_b) such that $r_a = r_{a,p} + r_{a,c}$ and $r_b = r_{b,p} + r_{b,c}$ satisfying

$$r_{a,p} \geq 0, \tag{3.30}$$

$$r_{a,c} \geq 0, \tag{3.31}$$

$$r_{b,p} \geq 0, \tag{3.32}$$

$$r_{b,c} \geq 0, \tag{3.33}$$

$$r_{a,p} \leq (g_{aa} - g_{ab})^+, \tag{3.34}$$

$$r_{a,c} \leq \min(g_{aa}, g_{ab}), \tag{3.35}$$

$$r_{a,p} + r_{b,c} \leq \max(g_{aa} - g_{ab}, g_{ba}), \tag{3.36}$$

$$r_{a,p} + r_{a,c} + r_{b,c} \leq \max(g_{aa}, g_{ba}), \tag{3.37}$$

$$r_{b,p} \leq (g_{bb} - g_{ba})^+, \tag{3.38}$$

$$r_{b,c} \leq \min(g_{bb}, g_{ba}), \tag{3.39}$$

$$r_{b,p} + r_{a,c} \leq \max(g_{bb} - g_{ba}, g_{ab}), \tag{3.40}$$

$$r_{b,p} + r_{b,c} + r_{a,c} \leq \max(g_{bb}, g_{ab}). \tag{3.41}$$

The component-separated achievable regions reveals where and why opportunities for increased rate over orthogonalized schemes exist, however, a more concise set of inequalities is shown in (3.42)–(3.48) below for the deterministic IC. Notice that (3.42)–(3.48) approximates the Gaussian IC HK region (3.12)–(3.29). Whether the opportunity can be used is predicated on User a knowing that the opportunity exists.

$$r_a \leq g_{aa} \tag{3.42}$$

$$r_b \leq g_{aa} \tag{3.43}$$

$$r_a + r_b \leq (g_{aa} - g_{ba})^+ + \max(g_{bb}, g_{ba}) \tag{3.44}$$

$$r_a + r_b \leq (g_{bb} - g_{ab})^+ + \max(g_{aa}, g_{ab}) \tag{3.45}$$

$$r_a + r_b \leq \max(g_{ab}, (g_{aa} - g_{ba})^+) + \max(g_{ba}, (g_{bb} - g_{ab})^+) \tag{3.46}$$

$$2r_a + r_b \leq \max(g_{aa}, g_{ab}) + (g_{aa} - g_{ba})^+ + \max(g_{ba}, (g_{bb} - g_{ab})^+) \tag{3.47}$$

$$r_a + 2r_b \leq \max(g_{bb}, g_{ba}) + (g_{bb} - g_{ab})^+ + \max(g_{ab}, (g_{aa} - g_{ba})^+). \tag{3.48}$$

3.1.2 Outer Bounds

3.1.2.1 Virtual Z-Channel

In an IC, each transmitter is faced with two objectives:

- On the direct link, a transmitter seeks to adapt its signal to increase its rate (increase entropy) in the presence of an interference signal.
- On the out-going interference link, a transmitter seeks to minimize its impact (reduce entropy).

A simpler IC, known as the Z-channel, often provides clarity regarding these competing objectives by considering the effect of only one interference link. The relationship between the Z-channel and IC has been noted previously, e.g., in derivation of outer bounds [40]. However, instead of considering a single Z-channel, we go one step further by "unwrapping" the interference channel into a double Z-channel, so as to simultaneously consider effects of both outgoing and incoming interference for a series of users (Figure 3.2).

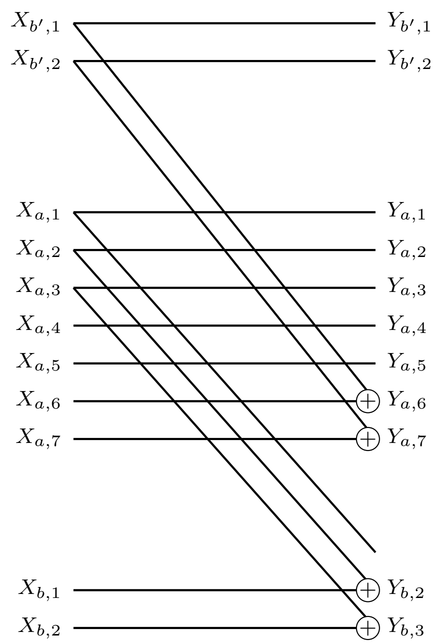


Figure 3.2: Deterministic IC of Figure 2.2(b) unwrapped into a double Z-channel.

In the case of a full view, the unwrapped IC does not supply any intuition beyond that given by known genie-aided bounds. However, a transmitter with a local view is uncertain of the state of at least one link in the unwrapped channel, and therefore must account for every possibility of unknown channel states and the resulting action of the other transmitter. Moreover, the most interfering input $\mathbf{X}_{b'}(\widehat{G}_{b'})$ at Receiver a may result from one local view at Transmitter b, $\widehat{G}_{b'}$, while the encoding function most sensitive to interference at Receiver b may result from a different local view, $\widehat{G}_b$.

Transmitter b faces similar challenges and expanding upon the notion of a limiting double Z-channel, we may visualize the constraints on policies resulting from local view IC as a series of virtual users arranged in a larger Z-channel with the following properties:

- A virtual User a always interferes with a virtual User b (and vice-versa), or does not interfere at all (terminates the Z-channel). The Z-channel may be cyclic.

- Each virtual transmitter uses the policy governing channel inputs corresponding to its local view.

- Any link gain known to both transmitters must be consistent throughout the virtual channel.

By visualizing the design of local view policies in an extended Z-channel, we can consider worst case virtual channel states, each of which corresponds to a sequence of policy responses to local views that are coupled.

3.1.2.2 Genie

The genie we describe here is unnecessary in the linear deterministic channel, owing to the fact that the channel is noiseless. However, the Gaussian IC genie is motivated by intuitions drawn from how, in the linear deterministic model, entropies of signals may be decomposed layer by layer through application of the chain rule, and examining the entropy of each signal level conditioned on higher levels. As an example, for the case shown below, we assume $g_{aa} \geq g_{ba} > 0$ (i.e., the direct link of a has a higher gain than the impinging interference from b) and consider the mutual information of Link a.

$$
\begin{aligned}
& I(\mathbf{X}_a^n; \mathbf{Y}_a^n) \\
& = H(\mathbf{Y}_a^n) - H(\mathbf{Y}_a^n | \mathbf{X}_a^n) && (3.49) \\
& = H(Y_{a,1}^n, \ldots, Y_{a,g_{aa}-g_{ba}}^n) + H(Y_{a,g_{aa}-g_{ba}+1}^n, \ldots, Y_{a,g_{aa}}^N | Y_{a,1}^n, \ldots, Y_{a,g_{aa}-g_{ba}}^n) \\
& \quad - H(X_{b,1}^n, \ldots, X_{b,g_{ba}}^n) && (3.50) \\
& = H(X_{a,1}^n, \ldots, X_{a,g_{aa}-g_{ba}}^n) + H(Y_{a,g_{aa}-g_{ba}+1}^n, \ldots, Y_{a,g_{aa}}^n | X_{a,1}^n, \ldots, X_{a,g_{aa}-g_{ba}}^n) \\
& \quad - H(X_{b,1}^n, \ldots, X_{b,g_{ba}}^n). && (3.51)
\end{aligned}
$$

Equation (3.50) results from an application of the chain rule, and (3.51) notes that the chain rule was applied at the boundary between interfered and uninterfered receive signal levels. If on the other hand $g_{ba} > g_{aa}$, we have

$$\begin{aligned} I(\mathbf{X}_a^n; \mathbf{Y}_a^n) = {} & H(X_{b,1}^n, \ldots, X_{b,g_{ba}-g_{aa}}^n) + H(Y_{a,g_{aa}-g_{ba}+1}^n, \ldots, Y_{a,g_{aa}}^n | X_{b,1}^n, \ldots, X_{b,g_{ba}-g_{aa}}^n) \\ & - H(X_{b,1}^n, \ldots, X_{b,g_{ba}}^n). \end{aligned} \tag{3.52}$$

Let

$$L_{a,i} \triangleq H(X_{a,i}^n | X_{a,1}^n \ldots, X_{a,i-1}^n), \tag{3.53}$$

$$L_{b,j} \triangleq H(X_{b,j}^n | X_{b,1}^n \ldots, X_{b,j-1}^n), \tag{3.54}$$

$$u_a^+ \triangleq (g_{aa} - g_{ba})^+, \tag{3.55}$$

$$u_a^- \triangleq (g_{ba} - g_{aa})^+. \tag{3.56}$$

Then (3.51) and (3.52) can be more generally bounded in (3.57) as

$$I(\mathbf{X}_a^n; \mathbf{Y}_a^n) \leq \left(n \min(g_{aa}, g_{ba}) - \sum_{k=1}^{g_{ba}} L_{b,k} \right) + \left(\sum_{i=1}^{u_a^+} L_{a,i} + \sum_{j=1}^{u_a^-} L_{b,j} \right). \tag{3.57}$$

Similarly, for Link b if

$$u_b^+ \triangleq (g_{bb} - g_{ab})^+, \tag{3.58}$$

$$u_b^- \triangleq (g_{ab} - g_{bb})^+, \tag{3.59}$$

then

$$I(\mathbf{X}_b^n; \mathbf{Y}_b^n) \leq \left(n \min(g_{bb}, g_{ab}) - \sum_{k=1}^{g_{ab}} L_{a,k} \right) + \left(\sum_{j=1}^{u_b^+} L_{b,j} + \sum_{i=1}^{u_b^-} L_{a,i} \right). \tag{3.60}$$

The first two expressions in both decompositions (3.57) and (3.60) emphasize that if the strengths of incoming signals are not equal, the most significant bits of the received signal are easy to decode, and the bottle neck occurs in those levels where the two signals overlap.

For the Gaussian IC, it is not apparent how to "decode the most significant bits" without restricting our analysis to layered coding schemes. However, if the upper levels of the signal (those modeled as non-interfered bits in the deterministic channel) are "easy to decode", then there should be little benefit in supplying these layers of the signal separately to the receiver. Therefore, we define a genie that provides a set of signals simulating the stronger of the two signals arriving at a noisy receiver. In constructing a series of such genie signals, we attempt to emulate the layering of new message content that is made explicit in the deterministic channel.

Our approach for constructing the genie is similar to [39] in the sense that each signal is derived from a series of degraded signals, each representing a component of a received signal that may be received in a potential channel state. Furthermore, like [39], our genie signals are conditionally (on the input) independent from the actual received signal.

Assume, for the explanation of the genie, that the Gaussian channel gains result in integer deterministic gains without the use of the floor function.

$$g_{aa} = \log\left(|h_{aa}|^2\right)^+, \tag{3.61}$$

$$g_{ab} = \log\left(|h_{ab}|^2\right)^+, \tag{3.62}$$

$$g_{ba} = \log\left(|h_{ba}|^2\right)^+, \tag{3.63}$$

$$g_{bb} = \log\left(|h_{bb}|^2\right)^+. \tag{3.64}$$

Let $Z_{a,\ell}^n \sim N(0,1)$ for $\ell \in \mathbb{N}$ be a series of length-n i.i.d. zero-mean complex Gaussian random vectors. We define the maximum number of signals derived from Transmitter a's input as

$$\ell_a^\star = \max\left(g_{aa}, g_{ab}\right), \tag{3.65}$$

and the signal $W^n_{a,\ell^\star_a}$ be given as

$$U^n_{a,\ell^\star_a} = \begin{cases} |h_{aa}|X^n_a + Z^n_{a,\ell^\star_a} & \text{if } |h_{aa}| \geq |h_{ab}| \\ |h_{ab}|X^n_a + Z^n_{a,\ell^\star_a} & \text{if } |h_{aa}| < |h_{ab}| \end{cases} \tag{3.66}$$

From $U^n_{a,\ell^\star_a}$, we define our a collection of serially degraded signals $\{W_{a,\ell}\}_{\ell\in\mathbb{N}}$

$$U^n_{a,\ell^\star_a-1} = U^n_{a,\ell^\star_a} + Z_{a,\ell^\star_a-1}, \tag{3.67}$$

$$U^n_{a,\ell^\star_a-2} = U^n_{a,\ell^\star_a-1} + \sqrt{2}Z_{a,\ell^\star_a-2}, \tag{3.68}$$

$$\vdots$$

$$U^n_{a,\ell} = U^n_{a,\ell-1} + \sqrt{2^{\ell^\star_a-\ell-1}}Z_{a,\ell}, \tag{3.69}$$

$$\vdots$$

$$U^n_{a,1} = \frac{1}{\sqrt{2}}U^n_{a,2} + \sqrt{2^{\ell^\star_a-2}}Z_{a,1}. \tag{3.70}$$

In each successive signal $U^n_{a,\ell}$ the power in the total noise term doubles. Additionally, the following Markov relationship is formed

$$U^n_{a,1} - U^n_{a,2} - \ \ldots\ - U^n_{a,\ell^\star_a} - X^n_a - Y^n_j, \tag{3.71}$$

where Y^n_j is either received signal. A similar collection of signals, $\{U_{b,\ell}\}_{\ell\in\mathbb{N}}$, is defined for Transmitter b's input as well.

To these signals, we add a phase correction term of the form

$$W^n_{aa,\ell} \triangleq \Phi_{aa}U^n_{a,\ell}, \tag{3.72}$$

$$W^n_{ab,\ell} \triangleq \Phi_{ab}U^n_{a,\ell}, \tag{3.73}$$

$$W^n_{ba,\ell} \triangleq \Phi_{ba}U^n_{b,\ell}, \tag{3.74}$$

$$W^n_{bb,\ell} \triangleq \Phi_{bb}U^n_{b,\ell}, \tag{3.75}$$

where $\Phi_{ij} = e^{j\angle h_{ij}}$ incorporates the appropriate phase into the genie signals.

At each receiver, the genie provides those signals that represent the "easy to decode" large scale variations. For instance, if $\|h_{aa}\| > \|h_{ba}\|$, then signals $\{W_{aa,\ell}\}_{\ell\in\{1,\ldots,g_{aa}-g_{ba}\}}$ are provided to Receiver a.

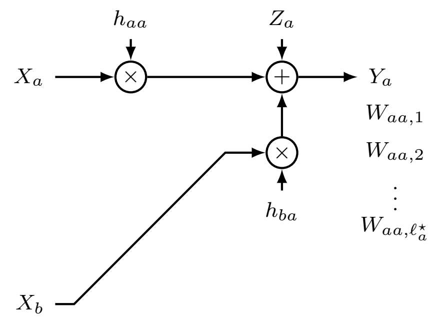


Figure 3.3: Genie for Receiver a in the Gaussian IC

Assuming $|h_{aa}| > |h_{ba}|$, our newly defined genie allows us to arrive at

$$
\begin{aligned}
I(X_a^n;Y_a^n) &\le I(X_a^n;Y_a^n, W_{aa,1}, W_{aa,2}, \ldots, W_{aa,u_a^+}) && (3.76)\\
&= h(Y_a^n, W_{aa,1}, W_{aa,2}, \ldots, W_{aa,u_a^+}) - h(Y_a^n, W_{aa,1}, W_{aa,2}, \ldots, W_{aa,u_a^+}|X_a^n) && (3.77)\\
&= h(Y_a^n|W_{aa,u_a^+}) - h(Y_a^n|W_{aa,u_a^+}, X_a^n) && (3.78)\\
&\quad + \sum_{\ell=1}^{u_a^+} h(W_{aa,\ell}|W_{aa,1}, \ldots, W_{aa,\ell-1}) - h(W_{aa,\ell}|W_{aa,1}, \ldots, W_{aa,\ell-1}, X_a^n) && (3.79)\\
&= h(Y_a^n|W_{aa,u_a^+}) - h(Y_a^n|W_{aa,u_a^+}, X_a^n) + \sum_{\ell=1}^{u_a^+} I(X_a^n; W_{aa,\ell}|W_{aa,1}, \ldots, W_{aa,\ell-1}). && (3.80)
\end{aligned}
$$

As desired, the expression (3.80) mimics (3.57) in its isolation of larger signal variations (more significant bits) from the variations which are contested by both direct and interference signals. Moreover, we notice that if h_{ai} was the channel gain with larger magnitude

$$
\begin{aligned}
&I(X_a^n; W_{ai,\ell}|W_{ai,1},\ldots,W_{ai,\ell-1}) \\
&\quad = I(X_a^n; W_{ai,\ell}|W_{ai,\ell-1}) && (3.81)\\
&\quad = h(W_{ai,\ell}|W_{ai,\ell-1}) - h(W_{ai,\ell}|X_a^n, W_{ai,\ell-1}) && (3.82)\\
&\quad = h(h_{ai}X_i^n + \sqrt{2^{\ell_a^\star-\ell}}Z^{n\prime}|h_{ai}X_i^n + \sqrt{2^{\ell_a^\star-\ell}}Z^{n\prime} + \sqrt{2^{\ell_a^\star-\ell}}Z^{n\prime\prime}) \\
&\qquad - h(h_{ai}X_i^n + \sqrt{2^{\ell_a^\star-\ell}}Z^{n\prime}|X_i^n, h_{ai}X_i + \sqrt{2^{\ell_a^\star-\ell}}Z^{n\prime} + \sqrt{2^{\ell_a^\star-\ell}}Z^{n\prime\prime}) && (3.83)\\
&\quad = h(h_{ai}X_i^n + \sqrt{2^{\ell_a^\star-\ell}}Z^{n\prime}|h_{ai}X_i^n + \sqrt{2^{\ell_a^\star-\ell}}Z^{n\prime} + \sqrt{2^{\ell_a^\star-\ell}}Z^{n\prime\prime}) \\
&\qquad - h(\sqrt{2^{\ell_a^\star-\ell}}Z^{n\prime}|\sqrt{2^{\ell_a^\star-\ell}}Z^{n\prime} + \sqrt{2^{\ell_a^\star-\ell}}Z^{n\prime\prime}) && (3.84)\\
&\quad = h(h_{ai}X_i^n + \sqrt{2^{\ell_a^\star-\ell}}Z^{n\prime}|h_{ai}X_i^n + \sqrt{2^{\ell_a^\star-\ell}}Z^{n\prime} + \sqrt{2^{\ell_a^\star-\ell}}Z^{n\prime\prime}) \\
&\qquad - \log\left(2\pi e\left[2^{\ell_a^\star-\ell-1}\right]\right) && (3.85)\\
&\quad = \log\left(1 + \frac{\|h_{ai}\|^2}{\|h_{ai}\|^2 + 2^{\ell_a^\star-\ell+1}}\right) \leq 1. && (3.86)
\end{aligned}
$$

As in the deterministic model, the payload of each genie signal level except for $\ell = 1$[1]

$$\Lambda_{a,\ell} \triangleq I(X_a^n; W_{ai,\ell}|W_{ai,1},\ldots,W_{ai,\ell-1}), \tag{3.87}$$

is constrained to at most 1 bit.

Additionally, we note that

$$
\begin{aligned}
\sum_{\ell=k}^{K} \Lambda_{a,\ell} &= \sum_{\ell=k}^{K} I(X_a^n; W_{ai,\ell}|W_{ai,1},\ldots,W_{ai,\ell-1}) && (3.88)\\
&= I(X_a^n; W_{ai,K}|W_{ai,1},\ldots,W_{ai,k}), && (3.89)
\end{aligned}
$$

is the point-to-point rate of the k^{th} through K^{th} layers. If we consider the interference-

[1] The topmost layer of the signal $\ell = 1$ is not subject to conditioning and thus is actually bounded by $\log(3)$.

noise term of (3.80), we have

$$h(Y_a^n|W_{aa,u_a^+}, X_a^n) = h(h_{ba}X_b^n + Z_a^n) \tag{3.90}$$

$$= I(X_b^n; W_{ba,g_{ba}}) + h(Z_a^n) \tag{3.91}$$

$$= I(X_b^n; W_{ba,g_{ba}}) + n\log(2\pi e). \tag{3.92}$$

If only a portion of the interfering signal is considered (i.e., if $g_{ba} > g_{aa}$ and the genie supplies Receiver a with layers from Transmitter b), we can also say

$$h(Y_a^n|W_{ba,u_a^-}, X_a^n) = h(h_{ba}X_b^n + Z_a^n|W_{ba,u_a^-}) \tag{3.93}$$

$$= I(X_b^n; W_{ba,g_{ba}}|W_{ba,u_a^-}) + n\log(2\pi e). \tag{3.94}$$

Consequently, the genie-aided decomposition of mutual information at each receiver can be bounded by

$$\begin{aligned} I(X_a^n; Y_a^n) \leq & \left(h(Y_a^n|W_{aa,u_a^+}, W_{ba,u_a^-}) - n\log(2\pi e) - \sum_{\ell=1}^{g_{ba}} \Lambda_{b,\ell} \right) \\ & + \left(\sum_{\ell=1}^{u_a^+} \Lambda_{a,\ell} + \sum_{\ell=1}^{u_a^-} \Lambda_{b,\ell} \right), \end{aligned} \tag{3.95}$$

$$\begin{aligned} I(X_b^n; Y_b^n) \leq & \left(h(Y_b^n|W_{bb,u_b^+}, W_{ab,u_b^-}) - n\log(2\pi e) - \sum_{\ell=1}^{g_{ab}} \Lambda_{a,\ell} \right) \\ & + \left(\sum_{\ell=1}^{u_b^+} \Lambda_{b,\ell} + \sum_{\ell=1}^{u_b^-} \Lambda_{a,\ell} \right), \end{aligned} \tag{3.96}$$

where $W_{ij,0}$ for $i, j \in \{a, b\}$ exist only as dummy (independent of the system or constant) signals. The expressions (3.95) and (3.96) will later be used in Section 3.4 to prove extensions of results for the linear deterministic IC to results for Gaussian IC. The similarities between the decoupling of bit layers in the deterministic IC and genie-aided decomposition for the Gaussian IC will permit similar analysis for both while facilitating accounting for the loss between the two models.

3.2 Minimum Performance Criterion

In order to simplify the formulation of the problem, we use the set of points $\overline{\mathcal{R}}_G^{\mathsf{TDM}}$ given by non-negative rate pairs satisfying

$$r_a = \tau \log\left(1 + |h_{aa}|^2\right) \tag{3.97}$$

$$r_b = (1-\tau) \log\left(1 + |h_{bb}|^2\right), \tag{3.98}$$

to define the minimum performance criterion for the LV-GIC. Note that this set of rate pairs lies between the boundary of the associated LDIC TDM and the Pareto optimal frontier of the TDM rate region with power scaling ($\mathcal{R}_G^{\mathsf{TDM}}$). The Gaussian IC minimum performance criterion can be more simply stated as

$$\frac{r_a(\widehat{H}_a)}{\log\left(1 + |h_{aa}|^2\right)} + \frac{r_b(\widehat{G}_b)}{\log\left(1 + |h_{bb}|^2\right)} \geq 1. \tag{3.99}$$

For the linear deterministic IC, we use the set of points $\overline{\mathcal{R}}_{LD}^{\mathsf{TDM}}$ given by non-negative rate pairs satisfying

$$r_a = \tau g_{aa} \tag{3.100}$$

$$r_b = (1-\tau) g_{bb}. \tag{3.101}$$

For the linear deterministic model, the minimum performance criterion can be more simply stated as

$$\frac{r_a(\widehat{G}_a)}{g_{aa}} + \frac{r_b(\widehat{G}_b)}{g_{bb}} \geq 1. \tag{3.102}$$

3.3 Results for the Linear Deterministic IC

In this section we state results for the linear deterministic IC. The capacity regions under criterion (3.102) for each of the seven views shown in Figure 2.3 falls in one of two categories. In the first category, we have Views 1 and 2 which enable opportunistic HK codes, thereby achieving rates dominating $\mathcal{R}_D^{\mathsf{TDM}}$. In the second category, containing Views 3–7, to achieve any point in the TDM-criterion-satisfying capacity region, a TDM scheme is sufficient.

Before stating the results we draw the reader's attention to relationships between views shown in Figure 3.4. The chart from top to bottom displays views with decreasing knowledge; the top row contains only the complete view of channel state, the second row contains views with knowledge of three link gains, the third contains views with knowledge of two link gains, and finally the fourth is composed of the case where each transmitter only knows its direct link. A directed edge from one view structure to another visualizes the reduction in the local view by one particular link. Since a view at the head of the edge has only a subset of the knowledge available to the view at the tail, intuitively one might assume that, for a given channel state, the capacity region of the reduced knowledge case is bounded by that of its predecessor. This is indeed the case, and simplifies the process of analyzing the many local views. Consequently, we need only analyze Views 1, 2, and 3 in full detail, and subsequently apply the results to Views 4–7.

Our capacity region characterizations are expressed as parameterization based on potential policies, and highlight coupling of policy responses in different channel states. When known, we also include a more concise set of inequalities stemming from the union over all such policies. Proofs are relegated to the Appendices.

To further provide intuition as to why each view either enables or inhibits opportunities for advanced transmission schemes, for Views 1–3 we use the deterministic IC shown in Figure 2.2(b) and either define a policy that performs uniformly better

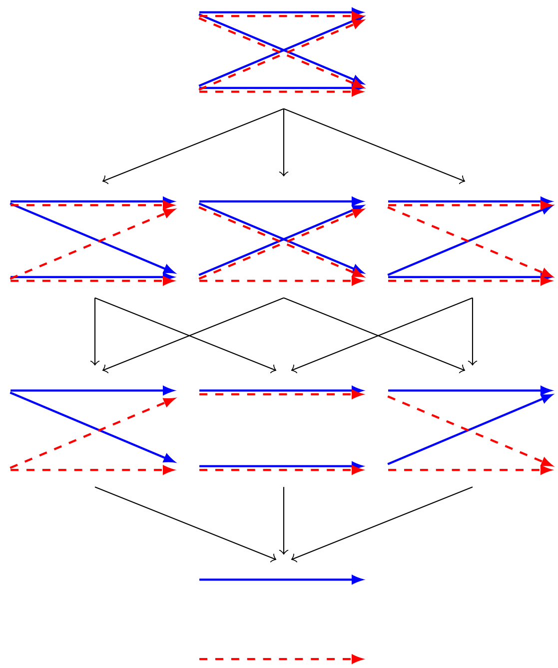


Figure 3.4: Decreasing Local Views: The chart depicts bounding relationships between views. An arrow from one view to another signifies that the performance of the first can be used to bound the other, since ignoring available knowledge is an option of any policy.

than TDM, or demonstrate why outperforming TDM is impossible.

3.3.1 Opportunity-Enabling Local Views

3.3.1.1 View 1

The local view capacity region for View 1 may exceed TDM, however the achievable region may be coupled across many states.

Theorem 3 (View 1 LV-LDIC Capacity Region)**.** *Let* $\widehat{G}_a = (g_{aa}, g_{ab}, \varnothing, g_{bb})$ *and* $\widehat{G}_b = (g_{aa}, \varnothing, g_{ba}, g_{bb})$ *and WLOG let* $g_{aa} \geq g_{bb}$*, and define for a specific channel state* $G = (g_{aa}, g_{ab}, g_{ba}, g_{bb})$ *the following value*

$$\delta = g_{aa} - g_{bb}. \tag{3.103}$$

The MPC-satisfying capacity region, $\mathcal{C}_{LD,1}$*, is the closure of the union over all regions* *indexed by nonnegative values* $\tau(g_{aa}, g_{bb}) \in (0,1)$*, containing non-negative rate pairs* $(r_a(\widehat{G}_a), r_b(\widehat{G}_b))$ *satisfying*

$$r_a(\widehat{G}_a) = r_a^c(\widehat{G}_a) + r_a^p(\widehat{G}_a), \tag{3.104}$$

$$r_b(\widehat{G}_b) = r_b^c(\widehat{G}_b) + r_b^p(\widehat{G}_b), \tag{3.105}$$

where

$$r_a\left(\widehat{G}_a\right) \le g_{aa} - g_{bb}\tau(g_{aa}, g_{bb}), \tag{3.106}$$

$$r_b\left(\widehat{G}_b\right) \le g_{aa}\tau(g_{aa}, g_{bb}), \tag{3.107}$$

$$r_a^c\left(\widehat{G}_a\right) \le \min_{\ell \ge 0}[\max(g_{bb} - \ell\delta, g_{ab}) + \ell\delta\tau(g_{aa}, g_{bb}) - g_{bb}\tau(g_{aa}, g_{bb})], \tag{3.108}$$

$$r_a^c\left(\widehat{G}_a\right) \le g_{ab}, \tag{3.109}$$

$$r_b^c\left(\widehat{G}_b\right) \le g_{aa}\tau(g_{aa}, g_{bb}), \tag{3.110}$$

$$r_b^c\left(\widehat{G}_b\right) \le \min_{\ell \ge 0}[\max(g_{ba} - \ell\delta, (g_{ba} - g_{aa})^+) + \ell\delta\tau(g_{aa}, g_{bb})], \tag{3.111}$$

$$r_a^c\left(\widehat{G}_a\right) + r_b\left(\widehat{G}_b\right) \le \min_{\ell \ge 0}[\max(g_{bb} - \ell\delta, g_{ab}) + \ell\delta\tau(g_{aa}, g_{bb})], \tag{3.112}$$

$$r_b^c\left(\widehat{G}_b\right) + r_a\left(\widehat{G}_a\right) \le \max(g_{ba}, g_{aa}), \tag{3.113}$$

$$\begin{aligned} r_b^c\left(\widehat{G}_b\right) + r_b\left(\widehat{G}_b\right) \le {} & \min_{\ell \ge 0}[\max(g_{ba} - (\ell+1)\delta, (g_{ba} - g_{aa})^+) \\ & + (g_{aa} + \ell\delta)\tau(g_{aa}, g_{bb})], \end{aligned} \tag{3.114}$$

$$r_a^c\left(\widehat{G}_a\right) + r_b^p\left(\widehat{G}_b\right) \le \min_{\ell \ge 0}[\max(g_{ab}, g_{bb} - g_{ba} - \ell\delta) + \ell\delta\tau(g_{aa}, g_{bb})], \tag{3.115}$$

$$\begin{aligned} \overline{r}_b^c\left(\widehat{G}_b\right) + r_a^p\left(\widehat{G}_a\right) \le {} & \min_{\ell \ge 0}[\max(g_{ba} - \ell\delta, (g_{aa} - g_{ab})^+, g_{ba} - g_{ab}, g_{ba} - g_{aa}) \\ & + \ell\delta\tau(g_{aa}, g_{bb})], \end{aligned} \tag{3.116}$$

$$r_a^p\left(\widehat{G}_a\right) \leq \min_{\ell\geq 0}[\max((g_{aa}-g_{ab})^+, g_{bb}-\ell\delta) - g_{bb}\tau(g_{aa},g_{bb}) + \ell\delta\tau(g_{aa},g_{bb})], \tag{3.117}$$

$$r_a^p\left(\widehat{G}_a\right) \leq (g_{aa}-g_{ab})^+, \tag{3.118}$$

$$r_b^p\left(\widehat{G}_b\right) \leq \min_{\ell\geq 0}[(g_{bb}-g_{ba}-\ell\delta)^+ + \ell\delta\tau(g_{aa},g_{bb})]. \tag{3.119}$$

Though the parameterized characterization of the region is somewhat unwieldy, each expression in (3.106)–(3.119) results from a particular class of virtual Z-channels. Moreover, minimization over ℓ_i in an expression actually describes at most two "worst-cases". Which of the two cases is truly worst depends on the value of $\tau_b(g_{aa}, g_{bb})$ and the channel state G.

Though it is possible to state the capacity region in a non-parametric form (through Fourier-Motzkin elimination or categorically for each of many different regimes), such a characterization does little to illuminate the form of the capacity region. We find it more illustrative to provide an example of a policy outperforming TDM in the example channel (Figure 3.5(a)). Transmitter b transmits at full rate ($r_b(\widehat{G}_b) = g_{bb} = 2$). Transmitter a uses a HK code where the common message has rate $r_{a,c} = g_{ab} - g_{bb} = 1$. *All three interfering layers* are used in a codebook drawn from a random distribution, which can be interpreted as the most significant bit carrying a one-bit message, and the next two layers providing parity. So far we have ensured decodability of Transmitter b's message at Receiver b.

A private message is encoded over all $g_{aa} - g_{ab} = 4$ private layers and rate $r_{a,p} = 2$. Regardless of the value of g_{ba} (which is unknown to Transmitter a) the component rates at Receiver a satisfy (3.30)–(3.37), so the rate point $(r_{a,c} + r_{a,p}, r_{b,c}) = (3, 2)$ is achievable. We see that

$$\begin{aligned}\frac{r_a(\widehat{G}_a)}{g_{aa}} + \frac{r_b(\widehat{G}_b)}{g_{bb}} &= \frac{3}{7} + \frac{2}{2} \\ &= \frac{10}{7} > 1,\end{aligned}$$

as desired.

3.3.1.2 View 2

For View 2, each transmitter is aware of which of its signal levels may be causing interference, and which may be interfered with. Thus, each transmitter may opportunistically align bits to appropriate signal levels. Note that in the parameterized characterization, the constraint on rate for each transmitter is independent of the unknown link gain. This is because the worst case(s) — which results in TDM even in the full view scenario — has already been considered, and additional bits are gained through opportunism.

Theorem 4 (View 2 LV-IC Capacity Region). *Let* $\widehat{G}_a = (g_{aa}, g_{ab}, g_{ba}, \varnothing)$ *and* $\widehat{G}_b = (\varnothing, g_{ab}, g_{ba}, g_{bb})$ *be given for channel state* $G = (g_{aa}, g_{ab}, g_{ba}, g_{bb})$*. The MPC-satisfying capacity region,* $\mathcal{C}_{LD,2}$*, is the closure of the union over all regions indexed by non-negative values* $\tau(g_{ab}, g_{ba}) \in (0, 1)$*, containing non-negative rate pairs* $(r_a(\widehat{G}_a), r_b(\widehat{G}_b))$ *satisfying*

$$r_a(\widehat{G}_a) \leq g_{aa}, \tag{3.120}$$

$$r_a(\widehat{G}_a) \leq (g_{aa} - g_{ab})^+ + g_{ab}\tau_a(g_{ab}, g_{ba}), \tag{3.121}$$

$$r_a(\widehat{G}_a) \leq (g_{aa} - g_{ba})^+ + g_{ba}\tau_a(g_{ab}, g_{ba}), \tag{3.122}$$

$$r_a(\widehat{G}_a) \leq (g_{aa} - g_{ab} - g_{ba})^+ + (g_{ab} + g_{ba})\tau_a(g_{ab}, g_{ba}), \tag{3.123}$$

$$r_b(\widehat{G}_b) \leq g_{bb}, \tag{3.124}$$

$$r_b(\widehat{G}_b) \leq (g_{bb} - g_{ba})^+ + g_{ba}\tau_b(g_{ab}, g_{ba}), \tag{3.125}$$

$$r_b(\widehat{G}_b) \leq (g_{bb} - g_{ab})^+ + g_{ab}\tau_b(g_{ab}, g_{ba}), \tag{3.126}$$

$$r_b(\widehat{G}_b) \leq (g_{bb} - g_{ab} - g_{ba})^+ + (g_{ab} + g_{ba})\tau_b(g_{ab}, g_{ba}). \tag{3.127}$$

In general, this region cannot be achieved with a simple orthogonalized scheme. The details of the scheme are more rigorously explained within the proof, but it is essentially a deterministic analogue of the approach used in [19]. Knowledge of the outgoing link enables each transmitter to split its message into a public and private component, where the public message is coded using channel inputs that interfere with the other transmission. The private message is sent on the remaining inputs of the direct link, essentially "hidden in the noise floor" of the other receiver. Each receiver treats the desired public and private messages and the public message of the other user as a virtual MAC, and jointly decodes the components.

The similarity to the result of [19] also extends to the non-parametric characterization of the region.

Corollary 5. *The View 2 capacity region consists of all non-negative rate points* *satisfying*

$$r_a(\widehat{G}_a) \le g_{aa}, \tag{3.128}$$

$$r_b(\widehat{G}_b) \le g_{bb}, \tag{3.129}$$

$$r_a(\widehat{G}_a) + r_b(\widehat{G}_b) \le (g_{aa} - g_{ba})^+ + \max(g_{bb}, g_{ba}), \tag{3.130}$$

$$r_a(\widehat{G}_a) + r_b(\widehat{G}_b) \le (g_{bb} - g_{ab})^+ + \max(g_{aa}, g_{ab}), \tag{3.131}$$

$$r_a(\widehat{G}_a) + r_b(\widehat{G}_b) \le \max(g_{ab}, g_{aa} - g_{ba}) + \max(g_{ba}, g_{bb} - g_{ab}), \tag{3.132}$$

$$\frac{g_{ab} + g_{ba}}{g_{ba}} r_a(\widehat{G}_a) + r_b(\widehat{G}_b) \le \frac{g_{ab}}{g_{ba}} \max(g_{aa}, g_{ba}) + (g_{aa} - g_{ba})^+ + \max(g_{ba}, g_{bb} - g_{ab}), \tag{3.133}$$

$$r_a(\widehat{G}_a) + \frac{g_{ab} + g_{ba}}{g_{ba}} r_b(\widehat{G}_b) \le \frac{g_{ab}}{g_{ba}} \max(g_{bb}, g_{ba}) + (g_{bb} - g_{ba})^+ + \max(g_{ba}, g_{aa} - g_{ab}), \tag{3.134}$$

$$\frac{g_{ab} + g_{ba}}{g_{ab}} r_a(\widehat{G}_a) + r_b(\widehat{G}_b) \le \frac{g_{ba}}{g_{ab}} \max(g_{aa}, g_{ab}) + (g_{aa} - g_{ab})^+ + \max(g_{ab}, g_{bb} - g_{ba}), \tag{3.135}$$

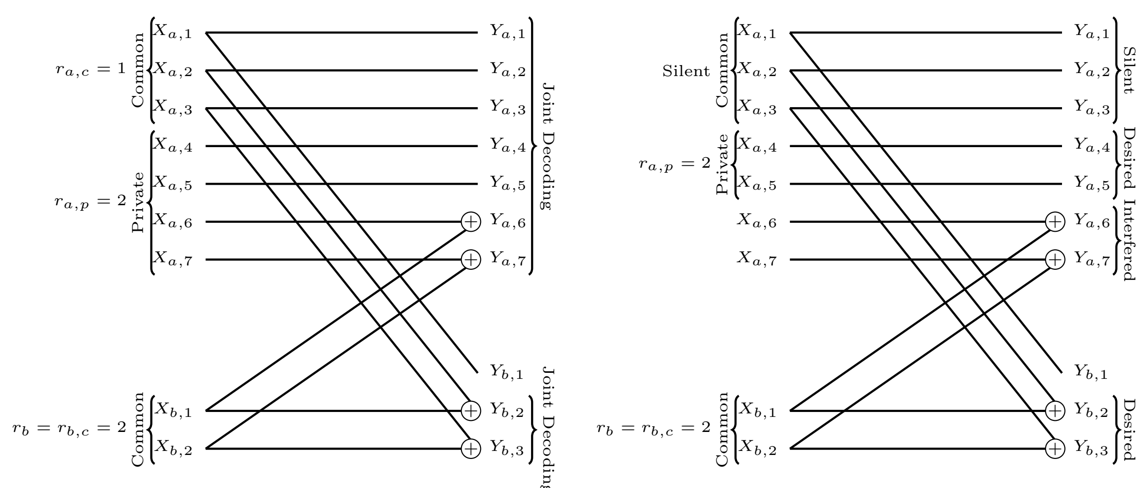
$$r_a(\widehat{G}_a) + \frac{g_{ab} + g_{ba}}{g_{ab}} r_b(\widehat{G}_b) \le \frac{g_{ba}}{g_{ab}} \max(g_{bb}, g_{ab}) + (g_{bb} - g_{ab})^+ + \max(g_{ab}, g_{aa} - g_{ba}). \tag{3.136}$$

The proof of the region specified in (3.128)–(3.136) is not included for brevity, however is easily derived by selecting linear combinations of expressions in Theorem 4 such that the τ terms are removed. As alluded to, (3.128)–(3.136) share many similarities with the full view capacity region ((3.42)–(3.48)). Specifically, (3.128)–(3.132) match (3.42)–(3.46) exactly. Of the remaining inequalities, if $g_{ab} = g_{ba}$, then (3.133) and (3.134) are equivalent to (3.135) and (3.136) as well as the inequalities (3.47) and (3.48) of the full view case. These inequalities are the *only reduction from the full view capacity region*, and if either $g_{ab} = g_{ba}$ or bounds (3.133)–(3.136) are dominated by the sum rate bounds, then the View 2 region and the full view region coincide. However, this does not imply a lack of operational loss between a full view and View 2. The parameterized characterization of Theorem 4 highlights the lack of system flexibility needed to achieve points within this region.

For an example of a policy outperforming TDM in the example channel, allow Transmitter b to always transmit at full rate ($r_b(\widehat{G}_b) = g_{bb} = 2$). Transmitter a uses a HK code, however the common message is constrained to rate $r_{a,c} = 0$. The private message is encoded over all the top two of the non-interfering signal levels ($X_{a,4}$ and $X_{a,5}$) at rate $r_{a,p} = 2$. Receiver a treats the interference as noise and decodes the private message. Consequently, we have

$$\begin{aligned} \frac{r_a(\widehat{G}_a)}{g_{aa}} + \frac{r_b(\widehat{G}_b)}{g_{bb}} &= \frac{2}{7} + \frac{2}{2} \\ &= \frac{9}{7} > 1, \end{aligned}$$

as desired. The coding scheme described is depicted below in Figure 3.5(b).



(a) View 1 Example

(b) View 2 Example

Figure 3.5: Examples of Opportunistic Schemes: Policy-dictated schemes responding to (a) View 1, and (b) View 2. In each, the view provides enough information to achieve a rate point outside the TDM rate region.

3.3.2 TDM-Optimal Local Views

In each of the remaining 5 views considered, performance uniformly better than TDM is not possible. However, we separate our statements of the remaining results to emphasize the distinction between views that still facilitate some degree of transmission coordination. Namely, Views 3 and 5 are able to capitalize on common knowledge of g_{aa} and g_{bb} in order to adjust which point on the TDM region boundary is used.

3.3.2.1 Views 3 & 5

Theorem 6 (Views 3 & 5 LV-IC Capacity Region)**.** *Let either* $\widehat{G}_a = (g_{aa}, \varnothing, g_{ba}, g_{bb})$ *and* $\widehat{G}_b = (g_{aa}, g_{ab}, \varnothing, g_{bb})$ *or* $\widehat{G}_a = (g_{aa}, \varnothing, \varnothing, g_{bb})$ *and* $\widehat{G}_b = (g_{aa}, \varnothing, \varnothing, g_{bb})$ *be given for channel state* $G = (g_{aa}, g_{ab}, g_{ba}, g_{bb})$*. The MPC-satisfying capacity regions,* $\mathcal{C}_{LD,3} = \mathcal{C}_{LD,5}$*, are the closure of the union over all regions indexed by nonnegative values* $\tau(g_{aa}, g_{bb}) \in (0, 1)$*, containing non-negative rate pairs* $(r_a(\widehat{G}_a), r_b(\widehat{G}_b))$ *satisfying*

$$r_a(\widehat{G}_a) \leq g_{aa}(1 - \tau(g_{aa}, g_{bb})), \tag{3.137}$$

$$r_b(\widehat{G}_b) \leq g_{bb}\tau(g_{aa}, g_{bb}). \tag{3.138}$$

Conversely, if the policies are such that there exists G *where*

$$\frac{r_a(\widehat{G}_a)}{g_{aa}} + \frac{r_b(\widehat{G}_b)}{g_{bb}} > 1 \tag{3.139}$$

then there also exists a G' *such that*

$$\frac{r_a(\widehat{G}_a)'}{g'_{aa}} + \frac{r_b(\widehat{G}'_b)}{g'_{bb}} < 1. \tag{3.140}$$

In other words, the capacity region is the same as what can be achieved by TDM,

$$\frac{r_a(\widehat{G}_a)}{g_{aa}} + \frac{r_b(\widehat{G}_b)}{g_{bb}} \leq 1, \tag{3.141}$$

with the only benefit of added information being that the exact time division, parameterized by$\tau(g_{aa}, g_{bb})$, can be chosen based on the direct link gains.

The criterion-satisfying capacity region for View 3 may be the most negative finding of this work. Despite *almost* complete knowledge of the network, transmitters are unable to perform better than TDM, which suggests that the costs paid by a transmitter to acquire knowledge of the incoming interference and the other direct link were wasted.

To see why this is the case, we again refer to the channel in Figure 2.2. First, we note that in order for a policy to outperform TDM, there must exist some τ^{min}, such that $r_a(\widehat{G}_a) \geq (1-\tau^{\mathsf{min}})g_{aa}$ and $r_b(\widehat{G}_b) \geq \tau^{\mathsf{min}} g_{bb}$.

Under View 3, each transmitter does not know its outgoing interference gain, but does know the direct link of the other link. Consider the POV of Transmitter a under the possibility $g'_{ab} = 1$ ($G' = (g_{aa}, g'_{ab} = 1, g_{ba}, g_{bb})$). From (3.60), we have

$$\begin{aligned}
n\tau^{\mathsf{min}} g_{bb} &\leq r_b(\widehat{G}_b^{(1)}) && (3.142)\\
&\leq I(\mathbf{X}_b^n; \mathbf{Y}_b^n) && (3.143)\\
&\leq n \min(g_{bb}, g_{ab}) - \sum_{k=1}^{g_{ab}} L_{a,k}(\widehat{G}_a) + \sum_{j=1}^{u_b^+} L_{b,j}(\widehat{G}_b^{(1)}) + \sum_{i=1}^{u_b^-} L_{a,i}(\widehat{G}_a) && (3.144)\\
&\leq n \min(2,1) - \sum_{k=1}^{1} L_{a,k}(\widehat{G}_a) + \sum_{j=1}^{1} L_{b,1}(\widehat{G}_b^{(1)}) && (3.145)\\
&\leq n - L_{a,1}(\widehat{G}_a) + L_{b,1}(\widehat{G}'_b). && (3.146)
\end{aligned}$$

Sweeping across a range of possible (from the POV of Transmitter a) channels, the set $\{1, 2, 3, 4, 5, 6, 7\}$, we have

$$2\tau^{\mathsf{min}} \leq \frac{1}{n}\left(L_{b,1}\left(\widehat{G}_b'\right) + n - L_{a,1}\left(\widehat{G}_a\right)\right), \tag{3.147}$$

$$2\tau^{\mathsf{min}} \leq \frac{1}{n}\left(2n - L_{a,1}\left(\widehat{G}_a\right) - L_{a,2}\left(\widehat{G}_a\right)\right), \tag{3.148}$$

$$2\tau^{\mathsf{min}} \leq \frac{1}{n}\left(2n - L_{a,2}\left(\widehat{G}_a\right) - L_{a,3}\left(\widehat{G}_a\right)\right), \tag{3.149}$$

$$2\tau^{\mathsf{min}} \leq \frac{1}{n}\left(2n - L_{a,3}\left(\widehat{G}_a\right) - L_{a,4}\left(\widehat{G}_a\right)\right), \tag{3.150}$$

$$2\tau^{\mathsf{min}} \leq \frac{1}{n}\left(2n - L_{a,4}\left(\widehat{G}_a\right) - L_{a,5}\left(\widehat{G}_a\right)\right), \tag{3.151}$$

$$2\tau^{\mathsf{min}} \leq \frac{1}{n}\left(2n - L_{a,5}\left(\widehat{G}_a\right) - L_{a,6}\left(\widehat{G}_a\right)\right), \tag{3.152}$$

$$2\tau^{\mathsf{min}} \leq \frac{1}{n}\left(2n - L_{a,6}\left(\widehat{G}_a\right) - L_{a,7}\left(\widehat{G}_a\right)\right). \tag{3.153}$$

The expressions (3.147)–(3.153) already suggest a microcosm of an inability to outperform TDM; every pair of signal levels consecutive in significance already simulate an orthogonalized scheme. By combining (3.147), (3.149), (3.151), and (3.153) gives us

$$\frac{1}{n}\sum_{i=1}^{7} L_{a,i}\left(\widehat{G}_a\right) \leq \frac{1}{n}L_{b,1}\left(\widehat{G}_b'\right) - \tau^{\mathsf{min}} + 7(1-\tau^{\mathsf{min}}), \tag{3.154}$$

and combining (3.148), (3.150), and (3.152) yields

$$\frac{1}{n}\sum_{i=1}^{6} L_{a,i}\left(\widehat{G}_a\right) \leq 6(1-\tau^{\mathsf{min}}). \tag{3.155}$$

If we recall that $n(1-\tau^{\mathsf{min}})g_{aa} \leq nr_a(\widehat{G}_a) \leq \sum_i L_{a,i}$, (3.155) becomes

$$\frac{1}{n}L_{b,1}\left(\widehat{G}_b'\right) \geq \tau^{\mathsf{min}}. \tag{3.156}$$

Since we have not used Transmitter a's knowledge of g_{ba}, (3.154)–(3.156) also hold for any other values of g_{ba} as well including $g_{ba}'' = 1$. Let $G'' = (g_{aa}, g_{ab}', g_{ba}'', g_{bb}) =$

$(7, 1, 1, 2)$. Then in another channel state, to decode reliably at Receiver a,

$$
\begin{aligned}
r_a(\widehat{G}''_a) &\leq I(\mathbf{X}_a; \mathbf{Y}_a) && (3.157)\\
&\leq \frac{1}{n}\sum_{i=1}^{6} L_{a,i}\left(\widehat{G}''_a\right) + 1 - \frac{1}{n}L_{b,1}\left(\widehat{G}'_b\right) && (3.158)\\
&\leq 6(1-\tau^{\mathsf{min}}) + 1 - \frac{1}{n}L_{b,1}\left(\widehat{G}'_b\right). && (3.159)
\end{aligned}
$$

Combining the fact $r_a(\widehat{G}''_a) \geq 7(1-\tau^{\mathsf{min}})$ and (3.156) implies $\frac{1}{n}L_{b,1}\left(\widehat{G}'_b\right) = \tau^{\mathsf{min}}$, which with (3.154) proves

$$
r_a(\widehat{G}_a) = 7(1-\tau^{\mathsf{min}}) = (1-\tau^{\mathsf{min}})g_{aa}. \tag{3.160}
$$

This example demonstrates how Transmitter a's inability to effectively align its interference signal prevents performance above that of TDM. A similar series of arguments confirms that Transmitter b also is limited to the TDM rate, however we omit this in lieu of the more general proof in the Appendix.

In View 5, transmitters have even less knowledge than in View 3, and thus is bounded by the same level of performance. However, it is interesting to note that the knowledge common to both transmitters is all the knowledge available to each transmitter. This not only allows them to synchronize their decision, but also suggests that View 5 models a centralized compound IC. It is therefore worth mentioning that the extension to multiple states discussed in [39] results in the same conclusion for the View 5 deterministic IC.

3.3.2.2 Views 4, 6, & 7

Theorem 7 (Views 4, 6, & 7 LV-IC Capacity Regions)**.** *Given any of the following three sets of local views*

- $\widehat{G}_a = (g_{aa}, \varnothing, g_{ba}, \varnothing)$ *and* $\widehat{G}_b = (\varnothing, g_{ab}, \varnothing, g_{bb})$

- $\widehat{G}_a = (g_{aa}, g_{ab}, \varnothing, \varnothing)$ *and* $\widehat{G}_b = (\varnothing, \varnothing, g_{ba}, g_{bb})$

- $\widehat{G}_a = (g_{aa}, \varnothing, \varnothing, \varnothing)$ *and* $\widehat{G}_b = (\varnothing, \varnothing, \varnothing, g_{bb})$,

for channel state $G = (g_{aa}, g_{ab}, g_{ba}, g_{bb})$. *The MPC-satisfying capacity regions,* $\mathcal{C}_{LD,4} = \mathcal{C}_{LD,6} = \mathcal{C}_{LD,7}$, *are given by the closure of the union over all regions indexed by nonnegative values* $\tau \in (0,1)$, *containing non-negative rate pairs* $(r_a(\widehat{G}_a), r_b(\widehat{G}_b))$ *satisfying*

$$r_a(\widehat{G}_a) \leq g_{aa}(1-\tau), \tag{3.161}$$

$$r_b(\widehat{G}_b) \leq g_{bb}\tau. \tag{3.162}$$

As we described while referencing Figure 3.4, policies relying on Views 4–7 can perform no better than views containing more information. Therefore, it is not surprising that no policy can outperform TDM in any of these views.

3.4 Results for the Gaussian IC

In this section, we extend our results to the Gaussian interference channel and characterize local view capacity regions to within a constant gap, and comment upon the local view GDoF of the Gaussian IC.

3.4.1 Main Result

Theorem 8 (Approximate Capacity Regions of Local View Gaussian ICs)**.** *Let*

$$g_{aa} = \lfloor \log\left(|h_{aa}|^2\right) \rfloor^+, \tag{3.163}$$

$$g_{ab} = \lfloor \log\left(|h_{ab}|^2\right) \rfloor^+, \tag{3.164}$$

$$g_{ba} = \lfloor \log\left(|h_{ba}|^2\right) \rfloor^+, \tag{3.165}$$

$$g_{bb} = \lfloor \log\left(|h_{bb}|^2\right) \rfloor^+, \tag{3.166}$$

and WLOG we assume $g_{aa} \geq g_{bb}$. For Local View k, the per-user gap between the MPC-satisfying Gaussian IC capacity region, $\mathcal{C}_{G,k}$, and the MPC-satisfying capacity region of a deterministic IC, $\mathcal{C}_{LD,k}$, with channel given by (3.163)–(3.166) is less than Δ_k bits where Δ_k is given in Table 3.1.

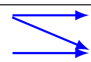
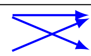
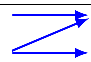
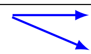
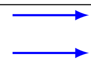
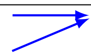
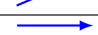
View (k)	View Diagram	Δ_k
1		$\begin{cases} \log(6) & \text{if } g_{aa} = g_{bb} \\ \log(9) + 2\max\left\{2\left\lceil \frac{g_{bb}}{g_{aa}-g_{bb}} \right\rceil + 1, \left\lceil \frac{g_{ba}}{g_{aa}-g_{bb}} \right\rceil + \left\lceil \frac{(g_{bb}-g_{ba})^+}{g_{aa}-g_{bb}} \right\rceil \right\} & \text{else} \end{cases}$
2		$2\log(6) + \log(3)$
3		$\left(\frac{\text{LCM}(g_{aa},g_{bb})}{g_{aa}} + \frac{\text{LCM}(g_{aa},g_{bb})}{g_{bb}} - 1\right)\log(6)$
4		$\log(6)$
5		$\left(\frac{\text{LCM}(g_{aa},g_{bb})}{g_{aa}} + \frac{\text{LCM}(g_{aa},g_{bb})}{g_{bb}} - 1\right)\log(6)$
6		$\log(6)$
7		$\log(6)$

Table 3.1: Per-user Gap Between Gaussian and Linear Deterministic MPC-Satisfying Capacity Regions

Detailed proofs for each view are located in Appendix A.3. In the remainder of this section we comment on the sources of gaps between models and intuitions that can be drawn from our result. Prior to explaining sources for gaps between the regions given in Theorems 3–7 and their Gaussian IC counterparts, we first clarify two minor details.

First, with regard to achievability of the LV Gaussian IC regions, for schemes prescribed for each local view linear deterministic IC (either TDM or a simple HK based code), there is a clear Gaussian IC analogue which is also achievable. For instance, the Gaussian TDM region contains the equivalent linear deterministic TDM region. Similarly, for the simple HK codes described in Section 3.1.1.2, the necessary conditions for achievability are looser for the Gaussian IC version. The main difference between HK schemes lies in generation of the codebook, where although we maintain respective splits between private and public components of the message, Gaussian

IC polices rely on codewords drawn from a Gaussian distribution (as opposed to the binary codewords of the deterministic IC).

Secondly, the exact Gaussian IC analogue of the deterministic channel TDM minimum performance criterion is not the Gaussian IC TDM criterion. In fact, the rates given by the deterministic model policies may in actuality not satisfy the Gaussian IC TDM criterion. However, because the deterministic IC TDM region boundary is interior to that of the TDM region of representative Gaussian ICs, the effective criterion used is a relaxation of the Gaussian IC TDM criterion, and its use does not reduce local view Gaussian IC capacity regions.

3.4.2 Approximate Capacity

To account for the gaps shown in Table 3.1, consider two main features of the linear deterministic approximation of the Gaussian channel: quantization of a complex gain h (real-valued magnitude) into an integer value g, and representation of a superposition of signals (i.e., addition) with modulo addition in the linear deterministic channel. We illustrate the impact of each by comparing the layer-by-layer decompositions of mutual information bounds for the deterministic and Gaussian ICs.

Consider the first term in both (3.57) and (3.95). In (3.57) the maximum entropy of the signal layers affected by both the desired and interference signals is no higher than if maximizing the entropy of just one of the two signals. On the other hand, the analogous term in the Gaussian IC is not so easily bounded. If $g_{aa} > g_{ba} > 0$ we have

$$
\begin{aligned}
& h\left(Y_a^n \middle| W_{aa,u_a^+}, W_{ba,u_a^-}\right) - n\log(2\pi e) \\
&= h\left(Y_a^n \middle| W_{aa,u_a^+}\right) - n\log(2\pi e) && (3.167) \\
&= h\left(h_{aa}X_a^n + h_{ba}X_b^n + Z_a^n \middle| \max\left(|h_{aa}|, |h_{ab}|\right)\Phi_{aa}X_a^n + \sqrt{2^{\max(g_{aa},g_{ab})-(g_{aa}-g_{ba})}}Z^{n\prime}\right) \\
&\quad - n\log(2\pi e) && (3.168) \\
&\leq h\left(h_{aa}X_a^n + h_{ba}X_b^n + Z_a^n - \frac{\max(|h_{aa}|, |h_{ab}|)\Phi_{aa}X_a^n + \sqrt{2^{g_{ba}+(g_{ab}-g_{aa})^+}}Z^{n\prime}}{\sqrt{2^{(g_{ab}-g_{aa})^+}}}\right) \\
&\quad - n\log(2\pi e) && (3.169) \\
&\leq h\left(\sqrt{2}\Phi_{aa}X_a^n + h_{ba}X_b^n + Z_a^n - \sqrt{2^{g_{ba}}}Z^{n\prime}\right) - n\log(2\pi e) && (3.170) \\
&\leq n\log\left(2\pi e\left[2 + \|h_{ba}\|^2 + 1 + 2^{g_{ba}}\right]\right) - n\log(2\pi e) && (3.171) \\
&\leq n\log\left(2\pi e\left[2^{g_{ba}+1} + 3 + 2^{g_{ba}}\right]\right) - n\log(2\pi e) && (3.172) \\
&\leq n\log\left(3(2^{g_{ba}}) + 3\right). && (3.173)
\end{aligned}
$$

If $g_{aa} < g_{ba}$ then

$$
\begin{aligned}
& h\Big(Y_a^n \Big| W_{aa,u_a^+}, W_{ba,u_a^-}\Big) - n\log(2\pi e) \\
&= h\left(Y_a^n \middle| W_{ba,u_a^-}\right) - n\log(2\pi e) && (3.174)\\
&= h\left(h_{aa}X_a^n + h_{ba}X_b^n + Z_a^n \middle| \max\left(|h_{bb}|, |h_{ba}|\right)\Phi_{ba}X_b^n + \sqrt{2^{\max(g_{bb},g_{ba})-(g_{ba}-g_{aa})}}Z^{n\prime}\right) \\
&\quad - n\log(2\pi e) && (3.175)\\
&\le h\left(h_{aa}X_a^n + h_{ba}X_b^n + Z_a^n - \frac{\max(|h_{bb}|,|h_{ba}|)\Phi_{ba}X_b^n + \sqrt{2^{g_{aa}+(g_{bb}-g_{ba})^+}}Z^{n\prime}}{\sqrt{2^{(g_{bb}-g_{ba})^+}}}\right) \\
&\quad - n\log(2\pi e) && (3.176)\\
&\le h\left(h_{aa}X_a^n + \sqrt{2}\Phi_{ba}X_b^n + Z_a^n - \sqrt{2^{g_{aa}}}Z^{n\prime}\right) - n\log(2\pi e) && (3.177)\\
&\le n\log\left(2\pi e\left[\|h_{aa}\|^2 + 2 + 1 + 2^{g_{aa}}\right]\right) - n\log(2\pi e) && (3.178)\\
&\le n\log\left(2\pi e\left[2^{g_{aa}+1} + 3 + 2^{g_{aa}}\right]\right) - n\log(2\pi e) && (3.179)\\
&\le n\log\left(3(2^{g_{aa}}) + 3\right). && (3.180)
\end{aligned}
$$

Finally if $g_{aa} = g_{ba}$ then

$$
\begin{aligned}
h(Y_a^n | W_{aa,u_a^+}, W_{ba,u_a^-}) - n\log(2\pi e) &= h(Y_a^n) - n\log(2\pi e) && (3.181)\\
&= h\left(h_{aa}X_a^n + h_{ba}X_b^n + Z_a^n\right) - n\log(2\pi e) && (3.182)\\
&\le n\log\left(\|h_{aa}\|^2 + \|h_{ba}\|^2 + 1\right) && (3.183)\\
&\le n\log\left(4(2^{g_{aa}}) + 1\right). && (3.184)
\end{aligned}
$$

We can upper bound the gap between this term and its deterministic channel counterpart over all channels:

$$\begin{aligned} h(Y_a^n|W_{aa,u_a^+}, W_{ba,u_a^-}) - n\log(2\pi e) - n\min\left(g_{aa}, g_{ba}\right) \\ \leq n\max\left[\log\left(3\left(2^{\min(g_{aa},g_{ba})}\right)+3\right), \log\left(4\left(2^{\min(g_{aa},g_{ba})}\right)+1\right)\right] \\ - n\min\left(g_{aa}, g_{ba}\right) \quad (3.185) \\ \leq n\log\left(6\right), \quad (3.186) \end{aligned}$$

which implies that in each interference scenario considered, there may be up to log(6) bits per channel use that is not utilized by rates prescribed by the linear deterministic capacity region.

This extra headroom is partially the result of the power gain (a multiple-access channel type of gain) that occurs when adding signals in the Gaussian model. Additionally, the quantized channel magnitudes in the deterministic model also incur a reduction in represented signal strength of both desired and interference signal components.

In the context of local view capacity analysis, this gap between the (tight) linear deterministic bound and our Gaussian outer bound exists for each interference scenario considered. Therefore, the larger the number of channel states that jointly constrain the rate of a transmitter (i.e., the number of virtual Z-channel links considered in establishing an outer bound) the larger the gap between linear deterministic and Gaussian capacity region boundaries. This is reflected in Views 4, 6 and 7, where due to extremely limited knowledge, only a single channel state can be established as a “worst case”, and our gap is relatively small.

This bound is admittedly not tight, and for certain cases (Views 2 and 5) application of existing bounds ([19] and [39] respectively) may result in smaller gaps. However, our analysis emphasizes the intuition imparted by the linear deterministic channel applied in the local view setting.

3.4.3 Generalized Degrees of Freedom

In order to make comments with regard to the GDoF of local view ICs, we first note the following property regarding integer multiples channel gains of linear deterministic ICs. The claim ultimately results from the linearity of the model and can be verified by examination of expressions defining each region:

Property 1 (Integer Multiples of Channel States)**.** If all the channel gains of a channel state G' can be expressed as an integer multiple of another state, $G' = cG$ where c is a positive integer, then the capacity region of the G' is the integer multiple of the capacity region of G.

$$\mathcal{C}(G') = c\mathcal{C}(G). \tag{3.187}$$

With respect to the Gaussian channel, this property coupled with the gap analysis results allows us to comment on the generalized degrees of freedom (GDoF) region for each view:

Corollary 9 (Local View GDoF Regions)**.** *Let α_1, α_2, and α_3, be positive rational values. The GDoF of View k is given by*

$$\mathcal{D}_k(\alpha) = \left\{ \left(\frac{r_a}{g_{aa}}, \frac{r_b}{g_{bb}} \right) : (r_a, r_b) \in \mathcal{C}_{D,k}(G) \right\}, \tag{3.188}$$

where G is such that $\frac{g_{bb}}{g_{aa}} = \alpha_1$, $\frac{g_{ba}}{g_{aa}} = \alpha_2$, $\frac{g_{ab}}{g_{aa}} = \alpha_3$.

While this does not define the local view GDoF for all values of α, because the rationals are dense in the reals, the local view GDoF can be found for an arbitrarily precise approximation of the parameter α.

3.5 Remarks

Remark 1: We note that the case of View 7 is least surprising in that when only direct links are known to each transmitter, TDM is an optimal approach. This scenario describes the largest contingent of past wireless protocols, and to some extent matches the intuition behind design of stochastic medium-access protocols such as in WiFi, which seek to orthogonalize transmission in a distributed way.

Remark 2: View 3 presents our most alarming finding, wherein although each Transmitter knows three out of four link gains, the view offers little opportunity for capacity gain at least in the generalized degrees of freedom sense. Though the analysis of gap between Gaussian and linear deterministic models suggests a potential (inside the log) power gain, how such a gain might be implemented is not immediately apparent.

Remark 3: One of our major conclusions shows the critical importance of each transmitter knowing its outgoing interference link in order to use advanced coding schemes to achieve performance universally better than TDM. Not only does knowledge of this link allow each transmitter to properly identify the interference regime of its signal (weak, strong, very strong), but it enables the more complex coding mechanisms that provide a GDoF gain. However, we note that knowledge of the outgoing link alone was not sufficient to reveal opportunities for capacity gain. Both View 1 and View 2 are given knowledge of at least one other link to provide just enough information for the policies of the two users to be coordinated.

Remark 4: In arriving at our results we developed a number of bounding techniques that facilitate analysis of interference channels. Although some were designed with intent towards analyzing local view interference channels — e.g., virtual Z-channels and unwrapping of interference channels — other techniques — notably our layered genie — may prove useful in providing insight into other scenarios.

Chapter 4

Local View Interference Mitigation with Receiver Cooperation

In this chapter, we study the effect of receiver cooperation upon interference mitigation with local views. The receiver cooperation scenario is intended to model base station cooperation in cellular uplink, where the transmitters with local views represent mobile users. One can view base station cooperation in uplink as migrating the role of message decoder from the base station to a regional server.

Recall that in Chapter 3 we assumed all receivers decode coherently. We extend this assumption to the receiver cooperation case, meaning that the two base stations have all the channel knowledge required to decode coherently. As a result, the information shared on the modeled cooperative link is channel output information. From the point of view of transmitters (mobile users) the challenge is knowing enough about the channel to take advantage of the cooperative link.

4.1 Minimum Performance Criterion

We first clarify our notion of a TDM-based minimum performance criterion (MPC) within the context of cooperation with local views. In the version of TDM we consider, during a user's allotted time, the full resources of the network are available for transmission. As in Chapter 3, we do not account for the additional (but bounded) gain afforded by scaling of power.

With a full view of the channel state, the TDM region of the receiver cooperation (SIMO-MAC) case $\mathcal{R}_{G,R}^{\mathsf{TDM-E}}$ consists of all rate pairs (r_a, r_b) satisfying

$$r_a = \tau_a \log\left(1 + |h_{aa}|^2 + |h_{ab}|^2 + 2\,|h_{aa}|\,|h_{ab}|\right) \tag{4.1}$$

$$r_b = \tau_b \log\left(1 + |h_{bb}|^2 + |h_{ba}|^2 + 2\,|h_{bb}|\,|h_{ba}|\right), \tag{4.2}$$

with $\tau_a + \tau_b \leq 1$. Notice that with receive cooperation, receivers may beamform to increase the effective SNR.

When considering local views, it is possible that nodes may not know they can achieve all of (4.1)–(4.2). For example in (4.1), the full TDM rate is only possible if Transmitter a knows h_{ab}. This only holds for Views 1, 2, and 4. For these three we define a region of *view-enhanced TDM rates*, whose boundary $\overline{\mathcal{R}}_{GR}^{\mathsf{TDM-E}}$ is given by non-negative rate pairs (r_a, r_b) satisfying

$$r_a\left(\widehat{H}_a\right) = \tau \log\left(1 + |h_{aa}|^2 + |h_{ab}|^2 + 2\,|h_{aa}|\,|h_{ab}|\right) \tag{4.3}$$

$$r_b\left(\widehat{H}_b\right) = (1-\tau) \log\left(1 + |h_{bb}|^2 + |h_{ba}|^2 + 2\,|h_{bb}|\,|h_{ba}|\right). \tag{4.4}$$

Therefore for Views 1, 2, and 4 with receiver cooperation we use $\overline{\mathcal{R}}_{GR}^{\mathsf{TDM-E}}$ as the set of minimum performance criterion rate pairs. For all other views studied, the set of points $\overline{\mathcal{R}}_{G}^{\mathsf{TDM}}$ from equations (3.97)–(3.98) is used to define minimum performance.

4.2 SIMO Multiple Access

The receiver cooperation scenario transforms the effective topology of the network into a two-user 1×2 single-input multiple-output multiple access channel (SIMO-MAC). This scenarios is relatively easy to analyze since, the Gaussian multiple access channel does not demand sophisticated encoding schemes to achieve capacity. If the majority of computational complexity is pushed to the receiver side, then through random Gaussian codebooks and joint decoding the full MAC capacity region can be achieved [18]. The same holds true for the receiver cooperation scenario simulated by SIMO-MAC. The challenge with local views then lies only in knowing that opportunities are available for increased rate.

With full view, the capacity region of the SIMO-MAC $\mathcal{C}_G^{\mathsf{SIMO-MAC}}$ is the closure of the union of all rate pairs (r_a, r_b) satisfying

$$\begin{aligned} r_a &\leq \log\left(1 + |h_{aa}|^2 + |h_{ab}|^2 + 2\,|h_{aa}|\,|h_{ab}|\right) && (4.5) \\ r_b &\leq \log\left(1 + |h_{bb}|^2 + |h_{ba}|^2 + 2\,|h_{bb}|\,|h_{ba}|\right) && (4.6) \\ r_a + r_b &\leq \log\det\left(\mathbf{I} + \mathbf{H}\mathbf{H}^{\dagger}\right), && (4.7) \end{aligned}$$

where

$$\mathbf{H} = \begin{bmatrix} h_{aa} & h_{ba} \\ h_{ab} & h_{bb} \end{bmatrix}, \tag{4.8}$$

and $\dagger$ denotes the Hermitian transpose. The bound (4.7) can be further simplified to

$$\begin{aligned} r_a + r_b \leq\ & \log\left(1 + |h_{aa}|^2 + |h_{ab}|^2 + |h_{ba}|^2 + |h_{bb}|^2\right. \\ & \left. + |h_{aa}|^2\,|h_{bb}|^2 + |h_{ab}|^2\,|h_{ba}|^2 + h_{aa}^* h_{ab} h_{ba} h_{bb}^* + h_{aa} h_{ab}^* h_{ba}^* h_{bb}\right). \end{aligned} \tag{4.9}$$

These expressions and in particular (4.9) will be instrumental in characterizing what rates policies may achieve.

As a final note, and for comparison later with our results on the LV-GIC with receiver cooperation, the GDoF region may be defined as all nonnegative pairs (d_a, d_b)

satisfying the boundaries

$$d_a \leq \max\left\{1, \frac{\log\left(|h_{ab}|^2\right)}{\log\left(|h_{aa}|^2\right)}\right\} \tag{4.10}$$

$$d_b \leq \max\left\{1, \frac{\log\left(|h_{ba}|^2\right)}{\log\left(|h_{bb}|^2\right)}\right\} \tag{4.11}$$

$$\log\left(|h_{aa}|^2\right) d_a + \log\left(|h_{bb}|^2\right) d_b$$
$$\leq \begin{cases} \max\left\{\log\left(|h_{aa}|^2\right), \log\left(|h_{ab}|^2\right), \log\left(|h_{ba}|^2\right), \log\left(|h_{bb}|^2\right)\right\} & \text{if } \det \mathbf{H} = 0 \\ \max\left\{\log\left(|h_{aa}|^2 |h_{bb}|^2\right), \log\left(|h_{ab}|^2 |h_{ba}|^2\right)\right\} & \text{else} \end{cases} . \tag{4.12}$$

These expressions are computed directly from (4.5)–(4.7), but notice that in order to facilitate comparison with the non-cooperative case, we have normalized *not by the MAC single user capacities*, but rather by the single user capacities of the original interference channel; i.e., it is possible to achieve greater than one GDoF per user.

4.3 Results

In this section we present policies and the resulting rates regions for each LV-GIC with receiver cooperation, present the gap to capacity, as well as provide the GDoF characterization. Since we assume joint decoding at the receiver, the policy in each of these cases uses a random Gaussian codebook. Only the size or rate of the codebook used is determined by the policy.

Views 1 & 2

Notice that from the perspective of each transmitter, receiver cooperation makes Views 1 and 2 equivalent down to a relabeling of links. WLOG we present the results assuming View 1.

Theorem 10. *Let views* $\widehat{H}_a = (h_{aa}, h_{ab}, \varnothing, h_{bb})$ *and* $\widehat{H}_b = (h_{aa}, \varnothing, h_{ba}, h_{bb})$ *given for the channel* $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$*. The MPC-satisfying capacity region with receiver cooperation,* $\mathcal{C}^{\mathsf{RC}}_{G,1}$*, is given by the closure of the union over* $\tau \in (0,1)$ *of non-negative rate pairs* $(r_a(\widehat{H}_a), r_b(\widehat{H}_b))$ *satisfying*

$$r_a\left(\widehat{H}_a\right) \leq \log\left(1 + |h_{aa}|^2 + |h_{ab}|^2 + 2\,|h_{aa}|\,|h_{ab}|\right) \tag{4.13}$$

$$r_b\left(\widehat{H}_b\right) \leq \log\left(1 + |h_{bb}|^2 + |h_{ba}|^2 + 2\,|h_{bb}|\,|h_{ba}|\right) \tag{4.14}$$

$$r_a\left(\widehat{H}_a\right) + r_b\left(\widehat{H}_b\right) \leq \log\det\left(\mathbf{I} + \mathbf{H}\mathbf{H}^\dagger\right) \tag{4.15}$$

$$\begin{aligned} r_a\left(\widehat{H}_a\right) \leq\; & \log\left(1 + |h_{aa}|^2 + |h_{ab}|^2 + |h^\star_{ba}|^2 + |h_{bb}|^2\right. \\ & \left. + |h_{aa}|^2\,|h_{bb}|^2 + |h_{ab}|^2\,|h^\star_{ba}|^2 - 2\,|h_{aa}|\,|h_{ab}|\,|h^\star_{ba}|\,|h_{bb}|\right) \\ & - \tau\log\left(1 + |h_{bb}|^2 + |h^\star_{ba}|^2 + 2\,|h_{bb}|\,|h^\star_{ba}|\right) \end{aligned} \tag{4.16}$$

$$\begin{aligned} r_b\left(\widehat{H}_b\right) \leq\; & \log\left(1 + |h_{aa}|^2 + |h^\star_{ab}|^2 + |h_{ba}|^2 + |h_{bb}|^2\right. \\ & \left. + |h_{aa}|^2\,|h_{bb}|^2 + |h^\star_{ab}|^2\,|h_{ba}|^2 - 2\,|h_{aa}|\,|h^\star_{ab}|\,|h_{ba}|\,|h_{bb}|\right) \\ & - (1-\tau)\log\left(1 + |h_{aa}|^2 + |h^\star_{ab}|^2 + 2\,|h_{aa}|\,|h^\star_{ab}|\right), \end{aligned} \tag{4.17}$$

where $h^\star_{ba}$ *and* $h^\star_{ab}$ *minimize the right hand side of (4.16) and (4.17) respectively.*

Notice first that (4.13)–(4.15) are boundaries of the full view capacity region. On the other hand, the expressions (4.16) and (4.17) represent the loss with respect to the full view capacity region resulting from Transmitter a not knowing h_{ba} and Transmitter b not knowing h_{ab}.

Additionally, notice that (4.16) and (4.17) are defined as minimizations over potential values of each transmitter's unknown link gain.

When we analyze the GDoF, it turns out the worst case value becomes the channel that results in a rank-1 matrix. This is evidenced in expression (4.21) of the following corollary, which scales and combines the GDoF equivalents of (4.16) and (4.17) so as to provide a characterization sans time-sharing parameter τ.

Corollary 11. *The GDoF region of View 1, $\mathcal{D}_1^{\mathsf{RC}}$, is the set of all nonnegative pairs (d_a, d_b) satisfying*

$$\frac{d_a}{\max\left\{1, \frac{\log\left(|h_{ab}|^2\right)}{\log\left(|h_{aa}|^2\right)}\right\}} \leq 1 \tag{4.18}$$

$$\frac{d_b}{\max\left\{1, \frac{\log\left(|h_{ba}|^2\right)}{\log\left(|h_{bb}|^2\right)}\right\}} \leq 1 \tag{4.19}$$

$$\begin{aligned}&\log\left(|h_{aa}|^2\right) d_a + \log\left(|h_{bb}|^2\right) d_b \\ &\quad\leq \begin{cases} \max\left\{\log\left(|h_{aa}|^2\right), \log\left(|h_{ab}|^2\right), \log\left(|h_{ba}|^2\right), \log\left(|h_{bb}|^2\right)\right\} & \textit{if } \det \mathbf{H} = 0 \\ \max\left\{\log\left(|h_{aa}|^2 |h_{bb}|^2\right), \log\left(|h_{ab}|^2 |h_{ba}|^2\right)\right\} & \textit{else} \end{cases}\end{aligned} \tag{4.20}$$

$$\begin{aligned}&\frac{\log\left(|h_{aa}|^2\right)}{\max\left\{\log\left(|h_{bb}|^2\right), \log\left(\frac{|h_{aa}|^2|h_{bb}|^2}{|h_{ab}|^2}\right)\right\}} d_a + \frac{\log\left(|h_{bb}|^2\right)}{\max\left\{\log\left(|h_{aa}|^2\right), \log\left(\frac{|h_{aa}|^2|h_{bb}|^2}{|h_{ba}|^2}\right)\right\}} d_b \\ &\leq \frac{\max\left\{\log\left(|h_{aa}|^2\right), \log\left(|h_{ab}|^2\right), \log\left(|h_{bb}|^2\right), \log\left(\frac{|h_{aa}|^2|h_{bb}|^2}{|h_{ab}|^2}\right)\right\}}{\max\left\{\log\left(|h_{bb}|^2\right), \log\left(\frac{|h_{aa}|^2|h_{bb}|^2}{|h_{ab}|^2}\right)\right\}} \\ &+ \frac{\max\left\{\log\left(|h_{aa}|^2\right), \log\left(|h_{ba}|^2\right), \log\left(|h_{bb}|^2\right), \log\left(\frac{|h_{aa}|^2|h_{bb}|^2}{|h_{ba}|^2}\right)\right\}}{\max\left\{\log\left(|h_{aa}|^2\right), \log\left(\frac{|h_{aa}|^2|h_{bb}|^2}{|h_{ba}|^2}\right)\right\}} - 1.\end{aligned} \tag{4.21}$$

View 3

Unlike Views 1 and 2, in View 3 the link gain unknown to each transmitter prevents either transmitter from assuming the availability of any receiver beamforming. Therefore the requirements of the MPC in this case are reduced and we arrive at the capacity region given by the following theorem.

Theorem 12. *Let views* $\widehat{H}_a = (h_{aa}, \varnothing, h_{ba}, h_{bb})$ *and* $\widehat{H}_b = (h_{aa}, h_{ab}, \varnothing, h_{bb})$ *given for the channel* $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$*. The MPC-satisfying capacity region,* $\mathcal{C}^{\mathsf{RC}}_{G,3}$*, is given by the union over* $\tau \in (0,1)$ *of nonnegative rate pairs* $(r_a(\widehat{H}_a), r_b(\widehat{H}_b))$ *satisfying*

$$r_a\left(\widehat{H}_a\right) \leq \log\left(1 + |h_{aa}|^2\right) \tag{4.22}$$

$$r_b\left(\widehat{H}_b\right) \leq \log\left(1 + |h_{bb}|^2\right) \tag{4.23}$$

$$r_a\left(\widehat{H}_a\right) + r_b\left(\widehat{H}_b\right) \leq \log\det\left(\mathbf{I} + \mathbf{H}\mathbf{H}^{\dagger}\right) \tag{4.24}$$

$$\begin{aligned} r_a\left(\widehat{H}_a\right) \leq\ & \log\left(1 + \frac{|h_{aa}|^2}{1 + |h_{ba}|^2}\right) + \log\left(1 + |h_{bb}|^2 + |h_{ba}|^2\right) \\ & - \tau \log\left(1 + |h_{bb}|^2\right) \end{aligned} \tag{4.25}$$

$$\begin{aligned} r_b\left(\widehat{H}_b\right) \leq\ & \log\left(1 + \frac{|h_{bb}|^2}{1 + |h_{ab}|^2}\right) + \log\left(1 + |h_{aa}|^2 + |h_{ab}|^2\right) \\ & - (1-\tau) \log\left(1 + |h_{aa}|^2\right). \end{aligned} \tag{4.26}$$

The parametric characterizations in expressions (4.25) and (4.25) can be combined as

$$\frac{r_a\left(\widehat{H}_a\right)}{\log\left(1+|h_{bb}|^2\right)}+\frac{r_b\left(\widehat{H}_b\right)}{\log\left(1+|h_{aa}|^2\right)} \le \frac{\log\left(1+|h_{aa}|^2+|h_{ba}|^2+|h_{bb}|^2+\frac{|h_{aa}|^2|h_{bb}|^2}{1+|h_{ba}|^2}\right)}{\log\left(1+|h_{bb}|^2\right)} + \frac{\log\left(1+|h_{aa}|^2+|h_{ab}|^2+|h_{bb}|^2+\frac{|h_{aa}|^2|h_{bb}|^2}{1+|h_{ab}|^2}\right)}{\log\left(1+|h_{aa}|^2\right)} - 1, \tag{4.27}$$

by scaling each expression and summing. This leads to the following GDoF region through direct computation.

Corollary 13. *The GDoF region of View 3 with receiver cooperation, $\mathcal{D}_3^{\mathsf{RC}}$, is the set of all nonnegative pairs (d_a, d_b) satisfying*

$$d_a \le 1 \tag{4.28}$$

$$d_b \le 1 \tag{4.29}$$

$$\log\left(|h_{aa}|^2\right) d_a + \log\left(|h_{bb}|^2\right) d_b \le \begin{cases} \max\left\{\log\left(|h_{aa}|^2\right), \log\left(|h_{ab}|^2\right), \log\left(|h_{ba}|^2\right), \log\left(|h_{bb}|^2\right)\right\} & \textit{if } \det \mathbf{H} = 0 \\ \max\left\{\log\left(|h_{aa}|^2 |h_{bb}|^2\right), \log\left(|h_{ab}|^2 |h_{ba}|^2\right)\right\} & \textit{else} \end{cases} \tag{4.30}$$

$$\frac{\log\left(|h_{aa}|^2\right)}{\log\left(|h_{bb}|^2\right)} d_a + \frac{\log\left(|h_{bb}|^2\right)}{\log\left(|h_{aa}|^2\right)} d_b \le \frac{\max\left\{\log\left(|h_{aa}|^2\right), \log\left(|h_{ba}|^2\right), \log\left(|h_{bb}|^2\right), \log\left(\frac{|h_{aa}|^2|h_{bb}|^2}{|h_{ba}|^2}\right)\right\}}{\log\left(|h_{bb}|^2\right)} + \frac{\max\left\{\log\left(|h_{aa}|^2\right), \log\left(|h_{ab}|^2\right), \log\left(|h_{bb}|^2\right), \log\left(\frac{|h_{aa}|^2|h_{bb}|^2}{|h_{ab}|^2}\right)\right\}}{\log\left(|h_{aa}|^2\right)} - 1. \tag{4.31}$$

View 4

For View 4, we follow methods similar to Views 1 and 2, only for each transmitter the quality of the opposite direct link is unknown. Minimizing over this unknown, we arrive at the following.

Theorem 14. *Let views $\widehat{H}_a = (h_{aa}, h_{ab}, \varnothing, \varnothing)$ and $\widehat{H}_b = (\varnothing, \varnothing, h_{ba}, h_{bb})$ be given for the channel $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$. The MPC-satisfying capacity region with receiver cooperation, $\mathcal{C}^{\mathsf{RC}}_{G,4}$, is given by the closure of the union over $\tau \in (0,1)$ of non-negative rate pairs $(r_a(\widehat{H}_a), r_b(\widehat{H}_b))$ satisfying*

$$
\begin{aligned}
r_a\left(\widehat{H}_a\right) &\leq \log\left(1 + |h_{aa}|^2 + |h_{ab}|^2 + 2\,|h_{aa}|\,|h_{ab}|\right) && (4.32)\\
r_b\left(\widehat{H}_b\right) &\leq \log\left(1 + |h_{bb}|^2 + |h_{ba}|^2 + 2\,|h_{bb}|\,|h_{ba}|\right) && (4.33)\\
r_a\left(\widehat{H}_a\right) + r_b\left(\widehat{H}_b\right) &\leq \log\det\left(\mathbf{I} + \mathbf{H}\mathbf{H}^\dagger\right) && (4.34)\\
r_a\left(\widehat{H}_a\right) &\leq \log\left(1 + |h_{aa}|^2 + |h_{ab}|^2 + |h^\star_{ba}|^2 + |h^\star_{bb}|^2\right.\\
&\quad \left. + |h_{aa}|^2\,|h^\star_{bb}|^2 + |h_{ab}|^2\,|h^\star_{ba}|^2 - 2\,|h_{aa}|\,|h_{ab}|\,|h^\star_{ba}|\,|h^\star_{bb}|\right)\\
&\quad - \tau\log\left(1 + |h^\star_{bb}|^2 + |h^\star_{ba}|^2 + 2\,|h^\star_{bb}|\,|h^\star_{ba}|\right) && (4.35)\\
r_b\left(\widehat{H}_b\right) &\leq \log\left(1 + |h^\star_{aa}|^2 + |h^\star_{ab}|^2 + |h_{ba}|^2 + |h_{bb}|^2\right.\\
&\quad \left. + |h^\star_{aa}|^2\,|h_{bb}|^2 + |h^\star_{ab}|^2\,|h_{ba}|^2 - 2\,|h^\star_{aa}|\,|h^\star_{ab}|\,|h_{ba}|\,|h_{bb}|\right)\\
&\quad - (1-\tau)\log\left(1 + |h^\star_{aa}|^2 + |h^\star_{ab}|^2 + 2\,|h^\star_{aa}|\,|h^\star_{ab}|\right), && (4.36)
\end{aligned}
$$

where $h^\star_{bb}$ and $h^\star_{ba}$ minimize the right hand side of (4.35), and $h^\star_{aa}$ and $h^\star_{ab}$ minimize the right hand side of (4.36).

By assuming the unknown parameters mirror the known ones (e.g., for Transmitter a, $h^\star_{ba} = h_{aa}$ and $h^\star_{bb} = h_{ab}$) we may say that view-enhanced TDM, or receiver beamforming is a good approach to take:

Corollary 15. *The region given by* $\mathcal{R}_{GR}^{\mathsf{TDM-E}}$ *is within 1 bit per user of the MPC-satisfying capacity region of the LV-GIC with receiver cooperation.*

In other words, while a protocol that outperforms $\overline{\mathcal{R}}_{G,R}^{TDM-E}$ may exist, its benefit is bounded by two bits, and the local-view enhanced TDM that relies on receive beamforming is GDoF-optimal.

Corollary 16. *The GDoF region of View 4 with receiver cooperation,* $\mathcal{D}_4^{\mathsf{RC}}$*, is the set of all nonnegative pairs* (d_a, d_b) *satisfying*

$$\frac{d_a}{\max\left\{1, \frac{\log\left(|h_{ab}|^2\right)}{\log\left(|h_{aa}|^2\right)}\right\}} \leq 1 \tag{4.37}$$

$$\frac{d_b}{\max\left\{1, \frac{\log\left(|h_{ba}|^2\right)}{\log\left(|h_{bb}|^2\right)}\right\}} \leq 1 \tag{4.38}$$

$$\frac{d_a}{\max\left\{\frac{\log\left(|h_{ab}|^2\right)}{\log\left(|h_{aa}|^2\right)}, 1\right\}} + \frac{d_b}{\max\left\{\frac{\log\left(|h_{ba}|^2\right)}{\log\left(|h_{bb}|^2\right)}, 1\right\}} \leq 1. \tag{4.39}$$

While the TDM approach may be GDoF-optimal, it may still be possible to outperform the TDM scheme of the non-cooperative LV-GIC. In particular, if either $h_{ab} > h_{aa}$ or $h_{ba} > h_{bb}$ (i.e., strong or mixed interference) then this is true.

View 5

View 5 considers a special compound multiple access channel. The following result is derived from a straightforward minimization of the sum-rate constraint (4.9).

Theorem 17. *Let views* $\widehat{H}_a = (h_{aa}, \varnothing, \varnothing, h_{bb})$ *and* $\widehat{H}_b = (h_{aa}, \varnothing, \varnothing, h_{bb})$ *given for the channel* $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$*. The capacity region with receiver cooperation,* $\mathcal{C}_{G,5}^{\mathsf{RC}}$*, is given by the union over* $\tau \in [0,1]$ *of non-negative rate pairs* $(r_a(\widehat{H}_a), r_b(\widehat{H}_b))$

satisfying

$$r_a\left(\widehat{H}_a\right) \leq \log\left(1 + |h_{aa}|^2\right) \tag{4.40}$$

$$r_b\left(\widehat{H}_b\right) \leq \log\left(1 + |h_{bb}|^2\right) \tag{4.41}$$

$$r_a\left(\widehat{H}_a\right) + r_b\left(\widehat{H}_b\right) \leq \log\left(1 + |h_{aa}|^2 + |h_{bb}|^2 + 2\,|h_{aa}|\,|h_{bb}|\right) \tag{4.42}$$

First notice that because there is no mismatch between transmitter knowledge, a perfectly coordinated policy can be used, and the minimum performance criterion need not be enforced.

Additionally, we note that the expression (4.42) resembles that of the SISO-MAC, however with a slight gain in SNR. It can be shown that the effect of this gain is at most 1 bit, and consequently the GDoF region resembles exactly that of a SISO-MAC [8].

Corollary 18. *The GDoF region of View 5 with receiver cooperation, $\mathcal{D}_5^{\mathsf{RC}}$, is the set of all nonnegative pairs (d_a, d_b) satisfying*

$$d_a \leq 1 \tag{4.43}$$

$$d_b \leq 1 \tag{4.44}$$

$$\log\left(|h_{aa}|^2\right) d_a + \log\left(|h_{bb}|^2\right) d_b \leq \max\left\{\log\left(|h_{aa}|^2\right), \log\left(|h_{ab}|^2\right)\right\}. \tag{4.45}$$

View 6

Before considering the capacity region, we first propose the following achievable scheme for View 6, in which each transmitter essentially views the its local view as a SISO-MAC. Let $\tau \in [0, 1]$ be given.

Proposed Policy:

Rate Selection for $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$: For our proposed policy, when Transmitter a has the view $\widehat{H}_a = (h_{aa}, \varnothing, h_{ba}, \varnothing)$ or Transmitter b has the view $\widehat{H}_b = (\varnothing, h_{ab}, \varnothing, h_{bb})$, the rates of their codebooks satisfy

$$\begin{aligned} r_a\left(\widehat{H}_a\right) &\leq \log\left(1+|h_{aa}|^2\right) && (4.46)\\ r_b\left(\widehat{H}_b\right) &\leq \log\left(1+|h_{bb}|^2\right) && (4.47)\\ r_a\left(\widehat{H}_a\right)+r_b\left(\widehat{H}_b\right) &\leq \log\det\left(\mathbf{I}+\mathbf{H}\mathbf{H}^\dagger\right) && (4.48)\\ r_a\left(\widehat{H}_a\right) &\leq \max\left\{\log\left(1+|h_{aa}|^2\right), \log\left(1+|h_{ba}|^2\right)\right\} \\ &\quad -\tau\log\left(1+|h_{ba}|^2\right) && (4.49)\\ r_b\left(\widehat{H}_b\right) &\leq \max\left\{\log\left(1+|h_{bb}|^2\right), \log\left(1+|h_{ab}|^2\right)\right\} \\ &\quad -(1-\tau)\log\left(1+|h_{ab}|^2\right). && (4.50) \end{aligned}$$

Rate Selection for $H \neq (h_{aa}, h_{ab}, h_{ba}, h_{bb})$: When Transmitter a has the view $\widehat{H}_a = (h'_{aa}, \varnothing, h'_{ba}, \varnothing)$ with either $h'_{aa} \neq h_{aa}$ or $h'_{ba} \neq h_{ba}$, its codebook is constrained to rate less than or equal to $(1-\tau)\log\left(1+|h'_{aa}|^2\right)$. Similarly, when Transmitter b views $\widehat{H}_b = (\varnothing, h'_{ab}, \varnothing, h'_{bb})$ with either $h'_{bb} \neq h_{bb}$ or $h'_{ab} \neq h_{ab}$, its codebook is constrained to rate less than or equal to $\tau\log\left(1+|h'_{bb}|^2\right)$.

Now consider the MPC-satisfying capacity region:

Theorem 19. *Let views $\widehat{H}_a = (h_{aa}, \varnothing, h_{ba}, \varnothing)$ and $\widehat{H}_b = (\varnothing, h_a b, \varnothing, h_{bb})$ be given for the channel $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$. The MPC-satisfying capacity region with receiver cooperation, $\mathcal{C}^{\mathsf{RC}}_{G,6}$, is given by the closure of the union over $\tau \in (0,1)$ of non-negative* *rate pairs* $(r_a(\widehat{H}_a), r_b(\widehat{H}_b))$ *satisfying*

$$r_a\left(\widehat{H}_a\right) \leq \log\left(1+|h_{aa}|^2\right) \tag{4.51}$$

$$r_b\left(\widehat{H}_b\right) \leq \log\left(1+|h_{bb}|^2\right) \tag{4.52}$$

$$r_a\left(\widehat{H}_a\right)+r_b\left(\widehat{H}_b\right) \leq \log\det\left(\mathbf{I}+\mathbf{H}\mathbf{H}^\dagger\right) \tag{4.53}$$

$$r_a\left(\widehat{H}_a\right) \leq \log\left(1+\frac{|h_{aa}|^2}{1+|h_{ba}|^2}\right)+(1-\tau)\log\left(|h_{ba}|^2\right)+H_0(\tau) \tag{4.54}$$

$$r_b\left(\widehat{H}_b\right) \leq \log\left(1+\frac{|h_{bb}|^2}{1+|h_{ab}|^2}\right)+\tau\log\left(|h_{ab}|^2\right)+H_0(\tau), \tag{4.55}$$

where $H_0(p)$ *is the entropy function of the binary random variable with parameter* p*.*

At high SNR the our proposed policy and the capacity region are very similar in form, and in fact the gap between the two can be bounded.

Corollary 20. *The region given by* $\mathcal{R}_{G,6}^{\mathsf{RC}}$ *is within 2 bits per user of* $\mathcal{C}_{G,6}^{\mathsf{RC}}$*.*

More importantly, we are able to show that the constant gap implies that the approach proposed is GDoF-optimal.

Corollary 21. *The GDoF region of View 6 with receiver cooperation,* $\mathcal{D}_6^{\mathsf{RC}}$*, is the set of all nonnegative pairs* (d_a, d_b) *satisfying*

$$d_a \leq 1 \tag{4.56}$$

$$d_b \leq 1 \tag{4.57}$$

$$
\begin{aligned}
&\log\left(|h_{aa}|^2\right)d_a + \log\left(|h_{bb}|^2\right)d_b \\
&\quad \le \begin{cases} \max\left\{\log\left(|h_{aa}|^2\right), \log\left(|h_{ab}|^2\right), \log\left(|h_{ba}|^2\right), \log\left(|h_{bb}|^2\right)\right\} & \textit{if } \det \mathbf{H} = 0 \\ \max\left\{\log\left(|h_{aa}|^2 |h_{bb}|^2\right), \log\left(|h_{ab}|^2 |h_{ba}|^2\right)\right\} & \textit{else} \end{cases}
\end{aligned}
\tag{4.58}
$$

$$
\begin{aligned}
&\frac{\log\left(|h_{aa}|^2\right)}{\log\left(|h_{ba}|^2\right)}d_a + \frac{\log\left(|h_{bb}|^2\right)}{\log\left(|h_{ab}|^2\right)}d_b \\
&\qquad \le \max\left\{\frac{\log\left(|h_{aa}|^2\right)}{\log\left(|h_{ba}|^2\right)}, 1\right\} + \max\left\{\frac{\log\left(|h_{bb}|^2\right)}{\log\left(|h_{ab}|^2\right)}, 1\right\} - 1.
\end{aligned}
\tag{4.59}
$$

View 7

Recall that View 7 was the most knowledge limited scenario in the non-cooperative case, and this trend continues even in the presence of receiver cooperation. Though there may be an SNR gain, of all seven views considered in this document, View 7 is the only one that never benefits from cooperation in a GDoF sense.

Theorem 22. *Let views* $\widehat{H}_a = (h_{aa}, \varnothing, \varnothing, \varnothing)$ *and* $\widehat{H}_b = (\varnothing, \varnothing, \varnothing, h_{bb})$ *given for the channel* $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$*. The MPC-satisfying capacity region with receiver cooperation,* $\mathcal{C}^{\mathsf{RC}}_{G,7}$*, is given by the closure of the union over* $\tau \in (0,1)$ *of non-negative* *rate pairs* $(r_a(\widehat{H}_a), r_b(\widehat{H}_b))$ *satisfying*

$$r_a\left(\widehat{H}_a\right) \leq \log\left(1+|h_{aa}|^2\right) \tag{4.60}$$

$$r_b\left(\widehat{H}_b\right) \leq \log\left(1+|h_{bb}|^2\right) \tag{4.61}$$

$$r_a\left(\widehat{H}_a\right)+r_b\left(\widehat{H}_b\right) \leq \log\det\left(\mathbf{I}+\mathbf{H}\mathbf{H}^{\dagger}\right) \tag{4.62}$$

$$\begin{aligned} r_a\left(\widehat{H}_a\right) \leq\ & \log\left(1+|h_{aa}|^2+|h^{\star}_{bb}|^2+2\,|h_{aa}|\,|h^{\star}_{bb}|\right) \\ & -\tau\log\left(1+|h^{\star}_{bb}|^2\right) \end{aligned} \tag{4.63}$$

$$\begin{aligned} r_b\left(\widehat{H}_b\right) \leq\ & \log\left(1+|h_{bb}|^2+|h^{\star}_{aa}|^2+2\,|h_{bb}|\,|h^{\star}_{aa}|\right) \\ & -(1-\tau)\log\left(1+|h^{\star}_{aa}|^2\right), \end{aligned} \tag{4.64}$$

where $h^{\star}_{aa}$ *and* $h^{\star}_{bb}$ *minimize the right hand side of (4.63) and (4.64) respectively.*

From (4.63) and (4.64) if we select $h^{\star}_{aa} = h_{bb}$ and $h^{\star}_{bb} = h_{aa}$, we arrive at the following conclusion.

Corollary 23. *The region given by* $\mathcal{R}_G^{\mathsf{TDMA}}$ *is within 2 bits per user of the capacity region of the LV-GIC with receiver cooperation.*

Consequently, the GDoF region is no different than that of View 7 without cooperation.

Corollary 24. *The GDoF region of View 7 with receiver cooperation,* $\mathcal{D}_7^{\mathsf{RC}}$*, is the set of all nonnegative pairs* (d_a, d_b) *satisfying*

$$d_a \leq 1 \tag{4.65}$$

$$d_b \leq 1 \tag{4.66}$$

$$d_a + d_b \leq 1. \tag{4.67}$$

4.4 Remarks

Remark 1: Comparison of the GDoF regions[1] of receiver cooperation versus non-cooperation for the LV-GIC immediate demonstrates the power of cooperation in such networks. Whereas TDM was GDoF-optimal in all but two views (View 1 and 2) in the non-cooperative LV-GIC, only the most knowledge constrained case (View 7) shows no improvement. Moreover, the GDoF gains in many of the cases may be had for a wide variety of channel regimes.

Remark 2: When the full channel state is known, the multiplexing gain of the 1×2 SIMO-MAC can be shown to be 2. This implies that as SNR goes to infinity two spatial streams can coexist with both operating near their single user capacities. There exists an analogue in the GDoF approach, which can be most clearly seen in expression (4.12). If the known channel matrix has full rank, then a sum rate of at least

$$\log\left(|h_{aa}|^2\,|h_{bb}|^2\right) = \log\left(|h_{aa}|^2\right) + \log\left(|h_{bb}|^2\right) \tag{4.68}$$

can be approached, which approximates at high SNR the sum of the GIC single user capacities. With regard to the GDoF region, full utilization of spatial resources implies the GDoF region is box-shaped; i.e., there is no constraint on sum-GDoF.

In all of the views we considered, this was not the case. The *possibility* of a rank-1 channel matrix and satisfying the MPC for these rank-1 channels prevents full opportunistic usage of spatial resources. This implies: (1) Gains result more from a signal-space opportunism similar to that of the non-cooperative local view IC, and (2) full utilization of spectral *and* spatial resources requires full knowledge of the channel at both transmitters, a requirement that seems highly dubious given that the

[1]The GDoF region are better suited for comparison since our previous non-cooperative results provided approximate capacity regions, and GDoF better isolates the impact of interference from the impact of noise.

transmitters model mobile handheld devices.

Remark 3: Some of the views more accurately reflect the approach taken in current uplink architecture. View 7 in particular describes the nominal system where only the channel between one's mobile and associated base station is know. Unfortunately this view also offers no performance gain over non-cooperative approach.

On the other hand, View 4 describes the scenario where each mobile has knowledge of the link gain to each base station, a scenario that may occur frequently, particularly when the mobile is near the cell edge and must monitor nearby base station signals in the event of hand-off. View 4 at least offers the possible benefit of receive beamforming, which may provide significant boost in capacity in the strong interference regime. Since this regime is more likely to occur at the cell edge, the approach proposed in View 4 is an example of one that offers improvement while also being immediately realizable.

Remark 4: Though our problem has been cast as one of inter-cell interference. We also note that as base stations add physical and computational hardware to the tower, it is entirely possible that the SIMO-MAC topology may be used to describe the interactions within a single cell. If it were permitted to schedule users within a cell to the same time-frequency resources [36], then the analyses presented within this chapter apply to cases where each mobile has an incomplete measurement of its SIMO channel. That the analytical problem may be recast in such a way provides additional value to the impact of the results for future protocol design.

Chapter 5

Local View Interference Mitigation with Transmitter Cooperation

Unlike receiver cooperation, transmitter cooperation provides the focal point of the local view model (distributed transmitters) with the ability to share information. Many possible types of information to be passed on the cooperative link, however we focus on the two identified here.

One option is to share each transmitter's local view, thus providing each node with a more complete understanding of the network, and eliminating knowledge mismatch. The resulting model becomes a standard or compound (but centralized) interference channel.

Another type of information that may be shared on the cooperative link is message related data. In this scenario, mismatch in channel state still persists, and the node knowledge model is still local view, but transmitters are aware of aspects of the interfering signal, or may assist the transmission of the opposing stream. In this case, we interpret the network as a local view multiuser relay network, with multiple paths to each receiver.

In the scenario where sharing of both of the two types (channel state and message) of data are permitted, the resulting system is mathematically equivalent to a multiple-input single-output broadcast channel (MISO-BC), the related work of which we provide a brief overview in §5.2 of this chapter.

5.1 Minimum Performance Criterion

As in the receiver cooperation case, the possibility exists of nodes employing techniques such as beamforming, and thus we need to clarify the TDM-based minimum performance criterion. The boundary of view-enhanced TDM rates for transmitter cooperation $\overline{\mathcal{R}}_{GT}^{\mathsf{TDM-E}}$ is given by non-negative rate pairs $(r_a(\widehat{H}_a), r_b(\widehat{H}_a))$ satisfying

$$r_a\left(\widehat{H}_a\right) = \tau \log\left(1 + |h_{aa}|^2 + |h_{ba}|^2 + 2\,|h_{aa}|\,|h_{ba}|\right) \tag{5.1}$$

$$r_b\left(\widehat{H}_b\right) = (1-\tau) \log\left(1 + |h_{bb}|^2 + |h_{ab}|^2 + 2\,|h_{bb}|\,|h_{ab}|\right). \tag{5.2}$$

In the case of transmitter cooperation, this set represents phase coherent (but not power adjusted) transmit beamforming, and is used in the minimum performance criterion for Views 2, 3, and 6.

For all other views studied, the set of points $\overline{\mathcal{R}}_{G}^{\mathsf{TDM}}$ from equations (3.97)–(3.98) is used to define minimum performance.

5.2 MISO Broadcast

When full views and messages are shared between the transmitters, our two-user IC becomes equivalent to a 2×1 multiple-input single-output broadcast channel (MISO-BC) with a specific input covariance constraint. Since, in the non-cooperative case each transmitter obeyed a unit transmit power constraint, the analogue in the 2×1 MISO-BC is that input covariance matrix must have diagonal elements each less than one. In other words, the chosen input covariance matrix must lie within a set, $\mathcal{Q}$, denoting the set of positive semi-definite hermitian matrices, with diagonal elements less than or equal to one.

For a fixed set of link gains, $H = \{h_{aa}, h_{ab}, h_{ba}, h_{bb}\}$, the capacity region of the MISO-BC defines an absolute limit for either of the two transmitter cooperation approaches we consider. Moreover, the encoding techniques used to achieve capacity in Gaussian BC are considerably more sophisticated than in the SIMO-MAC studied in the previous chapter.

Gaussian broadcast channels were first studied in [16] as a single-input single-output (SISO) BC, and due the statistically degraded nature of the SISO physical model (one user has a lower SNR than the other), the capacity region of the SISO-BC was shown to be achieved using a technique known as superposition coding (SC). The SC approach to code design is similar to the simple HK codes discussed in §3.1.1.2, in that a message is split between multiple independent codebooks, and the codewords selected from each codebook are summed and transmitted. At the weaker receiver, one message is decoded treating the other as noise. At the stronger one, both are decoded (successively or jointly), and one message (that intended for the weaker receiver) is discarded. Therefore, in a sense the notion of the HK code common message is analogous to the weaker BC user, while the HK code private message corresponds to the message of the BC stronger user. For a degraded Gaussian MISO-BC ($h_{aa}h_{bb} - h_{ab}h_{ba} = 0$ and $|h_{aa}|^2 + |h_{ba}|^2 + 2|h_{aa}||h_{ba}| \geq |h_{bb}|^2 + |h_{ab}|^2 + 2|h_{bb}||h_{ab}|$) the SC region, $\mathcal{R}_G^{\mathsf{SC-TC}}$, is given by the union over $s \in [0,1]$ of all rate pairs (r_a, r_b) satisfying

$$r_a \leq \log\left(1 + s\left(|h_{aa}|^2 + |h_{ba}|^2 + 2|h_{aa}||h_{ba}|\right)\right) \tag{5.3}$$

$$r_b \leq \log\left(1 + \frac{(1-s)\left(|h_{bb}|^2 + |h_{ab}|^2 + 2|h_{bb}||h_{ab}|\right)}{1 + s\left(|h_{bb}|^2 + |h_{ab}|^2 + 2|h_{bb}||h_{ab}|\right)}\right). \tag{5.4}$$

Multiple antenna broadcast channels generally do not exhibit the degradedness that allows superposition to achieve capacity. The capacity of MIMO-BC (of which MISO-BC is a subset) was found recently [54], where it was shown that an approach known as successive dirty paper coding (DPC) achieves capacity [10]. In successive dirty paper coding, the first message is encoded with the intent that the signal of the second message is treated as noise at the first receiver. The second message is encoded using DPC [14], a specific form of Gelfand-Pinsker code [21] that applies in certain Gaussian channels, where the transmitter utilizes knowledge of the first message signal, treats it as a known interference sequence, and codes *around the interference.* It has been shown that at the second receiver, the effects of the interference from the first message can be completely mitigated, and the rate of the second message signal is then limited primarily by the power allocated to its transmission. For any Gaussian MISO-BC, the successive DPC region with the order a, b is given by the union over $s \in [0,1]$ of all rate pairs (r_a, r_b) satisfying

$$r_a \leq \log\left(1 + \frac{\begin{bmatrix} h_{aa} & h_{ba} \end{bmatrix} \mathbf{Q_a} \begin{bmatrix} h_{aa} & h_{ba} \end{bmatrix}^\dagger}{1 + \begin{bmatrix} h_{aa} & h_{ba} \end{bmatrix} \mathbf{Q_b} \begin{bmatrix} h_{aa} & h_{ba} \end{bmatrix}^\dagger}\right) \tag{5.5}$$

$$r_b \leq \log\left(1 + \begin{bmatrix} h_{ab} & h_{bb} \end{bmatrix} \mathbf{Q_b} \begin{bmatrix} h_{ab} & h_{bb} \end{bmatrix}^\dagger\right), \tag{5.6}$$

where $\mathbf{Q}_a$ and $\mathbf{Q}_b$ are input covariance matrices such that $\mathbf{Q}_a + \mathbf{Q}_b = \mathbf{Q} \in \mathcal{Q}$, and $.^\dagger$ represents the hermitian transpose operation.

The order of encoding may also be reversed, and one may also time share between orderings in order to arrive at the full range of rate pairs offered by DPC. The full region achievable by DPC was shown to be the capacity region of MISO-BC.

Both of SC and DPC, require considerable knowledge about the link gains. For instance, DPC requires knowledge of the interference sequence at one receiver. If one of the links to that receiver is unknown, then an optimal DPC can not be constructed. A SC designed under the assumption of a rank-1 channel may prove catastrophic when the assumption fails. Neither is particularly robust to the incomplete, mismatched channel knowledge of the local view model. This will be more apparent in the analysis of each individual LV-GIC with shared messages.

In deriving outer bounds on capacity satisfying minimum performance criterion, we find the following bound useful [52].

$$
\begin{aligned}
r_a + r_b &\leq \max_{\mathbf{Q} \in \mathcal{Q}} \log\det\left(\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^\dagger\right) && (5.7)\\
&\leq \log\left(1 + |h_{aa}|^2 + |h_{ab}|^2 + |h_{ba}|^2 + |h_{bb}|^2 + 2\,|h_{aa}|\,|h_{ba}| + 2\,|h_{ab}|\,|h_{bb}|\right.\\
&\qquad \left. + \,|h_{aa}h_{bb} - h_{ab}h_{ba}|^2\right). && (5.8)
\end{aligned}
$$

Notice the similarity to the SIMO-MAC sum-rate bound (4.9), with the main difference being the terms $2\,|h_{aa}|\,|h_{ba}|$ and $2\,|h_{bb}|\,|h_{ab}|$. These terms result from the possibility of correlation in transmitter signals, and did not feature in the MAC since channel inputs were independent across the two transmitters.

Using this outer bound and coding approaches such as superposition (if $\mathbf{H}$ is rank-1) or zero-forcing precoding ($\mathbf{H}$ is full rank), we are able to give the following characterization of the GDoF region of the LV-GIC with cooperative transmitters with full combined view and sharing of messages.

Corollary 25. *The GDoF region of the GIC with full view and shared messages is the set of pairs (d_a, d_b) satisfying*

$$
d_a \leq \max\left\{1, \frac{\log\left(|h_{ba}|^2\right)}{\log\left(|h_{aa}|^2\right)}\right\} \tag{5.9}
$$

$$
d_b \leq \max\left\{1, \frac{\log\left(|h_{ab}|^2\right)}{\log\left(|h_{bb}|^2\right)}\right\} \tag{5.10}
$$

$$\log\left(|h_{aa}|^2\right) d_a + \log\left(|h_{bb}|^2\right) d_b$$
$$\leq \begin{cases} \max\left\{\log\left(|h_{aa}|^2\right), \log\left(|h_{ab}|^2\right), \log\left(|h_{ba}|^2\right), \log\left(|h_{bb}|^2\right)\right\} & \textit{if } \det \mathbf{H} = 0 \\ \max\left\{\log\left(|h_{aa}|^2 |h_{bb}|^2\right), \log\left(|h_{ab}|^2 |h_{ba}|^2\right)\right\} & \textit{else} \end{cases}. \tag{5.11}$$

5.3 Sharing Local Views vs. Sharing Messages

Sharing Local Views

Consider again the local views shown in Figure 2.3. If we combine the known links of $\widehat{H}_a$ and $\widehat{H}_b$, then there are only two resulting cases to consider. Views 5 and 7 become the compound channel described by View 5, and all other local views become complete and matched, meaning the resulting system can be treated as an interference channel.

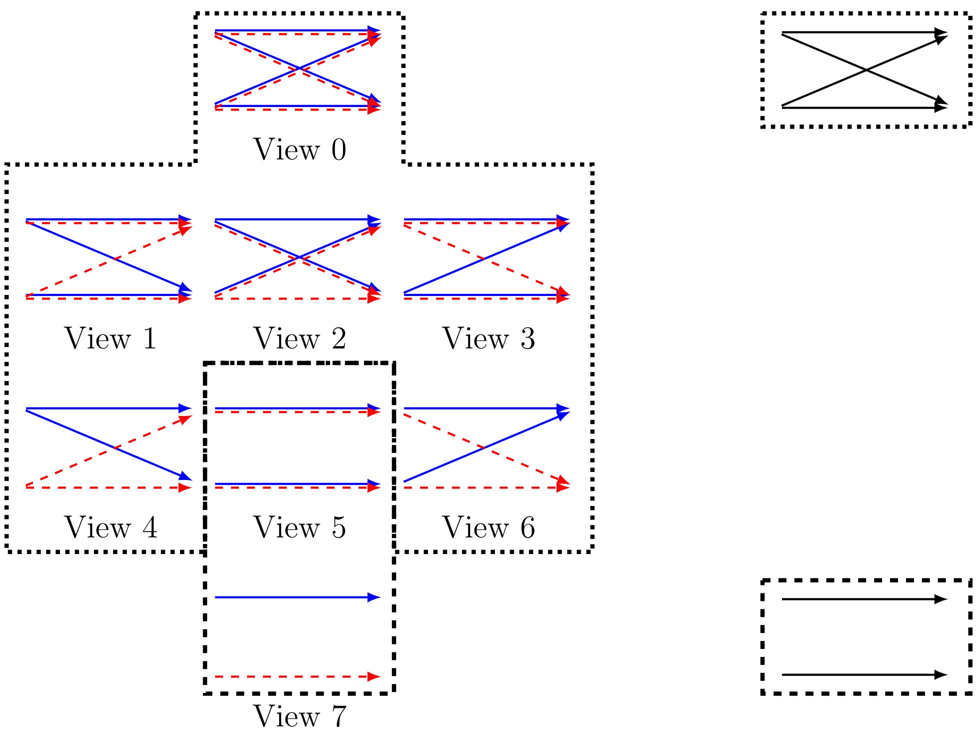


Figure 5.1: Sharing Local Views: After sharing local views, Views 1, 2, 3, 4, and 6 are equivalent to View 0, and Views 5 and 7 are equivalent to View 5.

The results of both these cases were covered in chapter 3, but to summarize, a gain over no cooperation only occurs if the knowledge known at each node includes at least either both incoming links, or both outgoing links. If this is true, then the view of the opposing user provides the complementary, unknown channel gains.

The practicality of base stations having these views is not addressed in detail in this dissertation, however we do note that in current practice, the mobiles (receivers) are more likely to track the signal strengths of nearby base stations signifying that perhaps Views 4, 6, and 7 can be accomplished either through a single round of training and feedback (View 4), or through monitoring of beacons (View 6). Implementing training across cells however is uncommon, and therefore, perhaps a method where mobiles monitor and report the signal strength of beacon signals is more feasible method of arriving at complementary views (albeit at longer time scales).

Sharing Messages

When only message data is shared on the cooperative link, the transmitters still may ave mismatched knowledge of channel state, and thus still make distributed decisions. The use of previously described approaches such as beamforming, zero-forcing, superposition codes, and dirty paper coding may become possible, if the local view provides enough knowledge of the channel to reliably use such a technique.

While we are as-yet unable to confirm whether such approaches may be used to provide a capacity gain over TDM for every local view, we present the following set of outer bounds on the minimum performance criterion satisfying capacity regions, which at least provide some intuition on the gains in performance message-only cooperation provides.

5.3.1 Capacity Outer Bounds for Message-Only Cooperation

View 1

In View 1, neither transmitter can rely on the possibility of transmit beamforming, since the second link (h_{ba} for Transmitter a) is unknown. Therefore, the MPC-based on $\overline{\mathcal{R}}_G^{TDM}$ is enforced, and we arrive at the following outer bound by minimizing over each transmitter's unknown parameter.

Proposition 26. *Let views $\widehat{H}_a = (h_{aa}, h_{ab}, \varnothing, h_{bb})$ and $\widehat{H}_b = (h_{aa}, \varnothing, h_{ba}, h_{bb})$ given for the channel $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$. The MPC-satisfying capacity region is a subset of the set $\overline{\mathcal{C}}_{G,1}^{\mathsf{TMC}}$ of rate pairs $(r_a(\widehat{H}_a), r_b(\widehat{H}_b))$ satisfying*

$$
\begin{aligned}
r_a\left(\widehat{H}_a\right) &\leq \log\left(1 + |h_{aa}|^2\right) && (5.12)\\
r_b\left(\widehat{H}_b\right) &\leq \log\left(1 + |h_{bb}|^2\right) && (5.13)\\
r_a\left(\widehat{H}_a\right) + r_b\left(\widehat{H}_b\right) &\leq \max_{\mathbf{Q}\in\mathcal{Q}} \log\det\left(\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^\dagger\right) && (5.14)\\
r_a\left(\widehat{H}_a\right) &\leq \log\left(1 + \frac{|h_{aa}|^2}{1 + |h_{ab}|^2}\right) + \log\left(1 + |h_{ab}|^2 + |h_{bb}|^2 + 2\,|h_{ab}|\,|h_{bb}|\right)\\
&\quad - \tau\log\left(1 + |h_{bb}|^2\right) && (5.15)\\
r_b\left(\widehat{H}_b\right) &\leq \log\left(1 + \frac{|h_{bb}|^2}{1 + |h_{ba}|^2}\right) + \log\left(1 + |h_{aa}|^2 + |h_{ba}|^2 + 2\,|h_{aa}|\,|h_{ba}|\right)\\
&\quad - (1-\tau)\log\left(1 + |h_{aa}|^2\right). && (5.16)
\end{aligned}
$$

View 2 & 3

View 2 and View 3 are equivalent down to a relabeling of links — both Transmitters know gains for both links terminating at the intended receiver, and only one link terminating at the unintended receiver. Therefore, we state the outer bound by again considering a similar set of outer bounds for the full view case, and minimizing over unknown parameters. WLOG we state the outer bound for View 3.

Proposition 27. *Let views $\widehat{H}_a = (h_{aa}, \varnothing, h_{ba}, h_{bb})$ and $\widehat{H}_b = (h_{aa}, h_{ab}, \varnothing, h_{bb})$ given for the channel $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$. The MPC-satisfying capacity region is a subset of the set $\overline{\mathcal{C}}_{G,3}^{\mathsf{TMC}}$ of rate pairs $(r_a(\widehat{H}_a), r_b(\widehat{H}_b))$ satisfying*

$$r_a\left(\widehat{H}_a\right) \leq \log\left(1 + |h_{aa}|^2 + |h_{ab}|^2 + 2\,|h_{aa}|\,|h_{ab}|\right) \tag{5.17}$$

$$r_b\left(\widehat{H}_b\right) \leq \log\left(1 + |h_{bb}|^2 + |h_{ba}|^2 + 2\,|h_{bb}|\,|h_{ba}|\right) \tag{5.18}$$

$$r_a\left(\widehat{H}_a\right) + r_b\left(\widehat{H}_b\right) \leq \max_{\mathbf{Q}\in\mathcal{Q}} \log\det\left(\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^\dagger\right) \tag{5.19}$$

$$\begin{aligned} r_a\left(\widehat{H}_a\right) \leq\ & \log\left(1 + |h_{aa}|^2 + |h^\star_{ab}|^2 + |h_{ba}|^2 + |h_{bb}|^2\right. \\ & + 2\,|h_{aa}|\,|h_{ba}| + 2\,|h_{bb}|\,|h^\star_{ab}| \\ & \left. + |h_{aa}|^2\,|h_{bb}|^2 + |h^\star_{ab}|^2\,|h_{ba}|^2 - 2\,|h_{aa}|\,|h^\star_{ab}|\,|h_{ba}|\,|h_{bb}|\right) \\ & - \tau \log\left(1 + |h_{bb}|^2 + |h^\star_{ab}|^2 + 2\,|h_{bb}|\,|h^\star_{ab}|\right) \end{aligned} \tag{5.20}$$

$$\begin{aligned} r_b\left(\widehat{H}_b\right) \leq\ & \log\left(1 + |h_{aa}|^2 + |h_{ab}|^2 + |h^\star_{ba}|^2 + |h_{bb}|^2\right. \\ & + 2\,|h_{aa}|\,|h^\star_{ba}| + 2\,|h_{bb}|\,|h_{ab}| \\ & \left. + |h_{aa}|^2\,|h_{bb}|^2 + |h_{ab}|^2\,|h^\star_{ba}|^2 - 2\,|h_{aa}|\,|h_{ab}|\,|h^\star_{ba}|\,|h_{bb}|\right) \\ & - (1-\tau) \log\left(1 + |h_{aa}|^2 + |h^\star_{ba}|^2 + 2\,|h_{aa}|\,|h^\star_{ba}|\right), \end{aligned} \tag{5.21}$$

where $h^\star_{ab}$ and $h^\star_{ba}$ minimize the right hand side of (5.20) and (5.21) respectively.

View 4

Like View 1, in View 4 neither transmitter can rely on the possibility of transmit beamforming, since the second link (h_{ba} for Transmitter a) is unknown and may be zero. Therefore, the MPC-based on $\overline{\mathcal{R}}_G^{TDM}$ is enforced, and we arrive at the following outer bound by minimizing over each transmitter's unknown parameters.

Proposition 28. *Let views $\widehat{H}_a = (h_{aa}, h_{ab}, \varnothing, \varnothing)$ and $\widehat{H}_b = (h_{aa}, \varnothing, hba, \varnothing)$ given for the channel $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$. The MPC satisfying capacity region is a subset of the set $\overline{\mathcal{C}}_{G,4}^{\mathsf{TMC}}$ of rate pairs $(r_a(\widehat{H}_a), r_b(\widehat{H}_b))$ satisfying*

$$r_a\left(\widehat{H}_a\right) \leq \log\left(1 + |h_{aa}|^2\right) \tag{5.22}$$

$$r_b\left(\widehat{H}_b\right) \leq \log\left(1 + |h_{bb}|^2\right) \tag{5.23}$$

$$r_a\left(\widehat{H}_a\right) + r_b\left(\widehat{H}_b\right) \leq \max_{\mathbf{Q}\in\mathcal{Q}} \log\det\left(\mathbf{I} + \mathbf{HQH}^\dagger\right) \tag{5.24}$$

$$\begin{aligned} r_a\left(\widehat{H}_a\right) \leq\ & \log\left(1 + \frac{|h_{aa}|^2}{1 + |h_{ab}|^2}\right) + \log\left(1 + |h_{ab}|^2 + |h_{bb}^\star|^2 + 2\,|h_{ab}|\,|h_{bb}^\star|\right) \\ & - \tau \log\left(1 + |h_{bb}^\star|^2\right) \end{aligned} \tag{5.25}$$

$$\begin{aligned} r_b\left(\widehat{H}_b\right) \leq\ & \log\left(1 + \frac{|h_{bb}|^2}{1 + |h_{ba}|^2}\right) + \log\left(1 + |h_{aa}^\star|^2 + |h_{ba}|^2 + 2\,|h_{aa}^\star|\,|h_{ba}|\right) \\ & - (1-\tau) \log\left(1 + |h_{aa}^\star|^2\right), \end{aligned} \tag{5.26}$$

where $h_{ab}^\star$ and $h_{ba}^\star$ minimize the right hand side of (5.25) and (5.26) respectively.

View 5

For View 5 we present an approach to transmission that is a bounded gap from the outer bound. The approach adopts a SC-based encoding and decoding, however encoding of the weaker user's message occurs at both transmitters *using independent but jointly typical codebooks*. Notice that the MPC does not come into play, since there exist no mismatch between transmitters. As such, this particular view can be considered as a special type of compound MISO-BC.

Policy Design

WLOG we assume $|h_{aa}| \geq |h_{bb}|$.

Rate Selection Assume the rate $r_b\left(\widehat{H}_b\right)$ is chosen, and may be expressed as

$$r_b\left(\widehat{H}_b\right) = \tau^\star \log\left(1 + |h_{bb}|^2\right) \tag{5.27}$$

for $\tau^\star \in [0,1]$. We select the rate, r_a, of User a assuming r_b is fixed, and satisfying

$$r_a\left(\widehat{H}_a\right) \leq \log\left(1 + |h_{aa}|^2\right) - \tau^\star \log\left(1 + |h_{bb}|^2\right). \tag{5.28}$$

Codebook Construction Transmitter b utilizes only a single-user code of typical sequences drawn from a complex Gaussian distribution with unit variance. Transmitter a uses a superposition scheme, where the first codebook (that meant to convey the message of Transmitter b) is composed of typical sequences drawn from a complex Gaussian distribution (independently of Transmitter b's codebook) with variance given by s satisfying

$$\log\left(1 + \frac{s\,|h_{aa}|^2}{1 + (1-s)\,|h_{aa}|^2}\right) = \tau^\star \log\left(1 + |h_{bb}|^2\right). \tag{5.29}$$

Moreover, given message b the codewords selected by Transmitter b and the first codebook of Transmitter a must be *jointly typical* of two independent Gaussian distributed variables.

The second codebook at Transmitter a is constructed from set of typical sequences drawn independently from a complex Gaussian distribution with variance $1-s$.

Decoding At both receivers the message of Transmitter b is decoded first, treating the signal bearing the message of Transmitter a as noise. At this point Receiver b is done, but Receiver a subtracts out the component of the received signal corresponding to message b, and then decodes message a. This is analogous to the decoding approach used in standard superposition codes, however encoding of the weaker link's message has occurred jointly.

Unlike the standard superposition approach for degraded MISO-BC, our approach explicitly does not make use of signal coherence, primarily in order to prevent the possibility of *destructive combining.* Thus, the message of Transmitter b is encoded independently at each transmitter and decoded jointly at both receivers. Consequently, our approach is robust to the unknown channel parameters

Lemma 29. *The region* $\mathcal{R}_{G,5}^{\mathsf{TC}}$ *containing the union over* $s \in [0,1]$ *of rate pairs* $(r_a(\widehat{H}_a), r_b(\widehat{H}_b))$ *satisfying*

$$r_a\left(\widehat{H}_a\right) \leq \log\left(1 + (1-s)\left|h_{aa}\right|^2\right) \tag{5.30}$$

$$r_b\left(\widehat{H}_b\right) \leq \min\left\{\log\left(1 + \frac{s\left|h_{aa}\right|^2}{1+(1-s)\left|h_{aa}\right|^2}\right), \log\left(1 + \left|h_{bb}\right|^2\right)\right\}. \tag{5.31}$$

is achievable for all channels H *resulting in local views* $\widehat{H}_a = (h_{aa}, \varnothing, \varnothing, h_{bb})$ *and* $\widehat{H}_b = (h_{aa}, \varnothing, \varnothing, h_{bb})$.

Proposition 30. *Let views* $\widehat{H}_a = (h_{aa}, \varnothing, \varnothing, h_{bb})$ *and* $\widehat{H}_b = (h_{aa}, \varnothing, \varnothing, h_{bb})$ *given for the channel* $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$. *The MPC satisfying capacity region is a subset of the set* $\overline{\mathcal{C}}_{G,5}^{\mathsf{TMC}}$ *of rate pairs* $(r_a(\widehat{H}_a), r_b(\widehat{H}_b))$ *satisfying*

$$r_a\left(\widehat{H}_a\right) \leq \log\left(1 + \left|h_{aa}\right|^2\right) \tag{5.32}$$

$$r_b\left(\widehat{H}_b\right) \leq \log\left(1 + \left|h_{bb}\right|^2\right) \tag{5.33}$$

$$r_a\left(\widehat{H}_a\right) + r_b\left(\widehat{H}_b\right) \leq \log\left(1 + 2\left(\left|h_{aa}\right|^2 + \left|h_{bb}\right|^2 + 2\left|h_{aa}\right|\left|h_{bb}\right|\right)\right) \tag{5.34}$$

Theorem 31. *Let views* $\widehat{H}_a = (h_{aa}, \varnothing, \varnothing, h_{bb})$ *and* $\widehat{H}_b = (h_{aa}, \varnothing, \varnothing, h_{bb})$ *given for the* *channel $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$. The Joint SC-based region of (5.30)–(5.31) is within 3 bits of the capacity region, $\mathcal{C}^{\mathsf{TC}}_{G,5}$, with message transmitter cooperation.*

View 6

In View 6, a situation arises analogous to that in View 4 with receiver cooperation. The possibility of a rank-deficient channel allows us to claim the following:

Theorem 32. *Let views $\widehat{H}_a = (h_{aa}, \varnothing, h_{ba}, \varnothing)$ and $\widehat{H}_b = (\varnothing, h_{ab}, \varnothing, h_{bb})$ given for the channel $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$. The region given by $\mathcal{R}^{\mathsf{TDM-E}}_{GT}$ is the MPC-satisfying capacity region, $\mathcal{C}^{\mathsf{TMC}}_{G,6}$, of the LV-GIC with message transmitter cooperation.*

View 7

As in View 6, with only View 7, the lack of any knowledge about the channel to the opposite receiver limits performance severely. In this case because even transmit beamforming cannot be assumed, there is no significant gain over the non-cooperative case.

Theorem 33. *Let views $\widehat{H}_a = (h_{aa}, \varnothing, \varnothing, \varnothing)$ and $\widehat{H}_b = (\varnothing, \varnothing, \varnothing, h_{bb})$ given for the channel $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$. The region given by $\mathcal{R}^{\mathsf{TDMA}}_{G}$ is within 3 bits per user of the MPC-satisfying capacity region, $\mathcal{C}^{\mathsf{TMC}}_{G,7}$, of the LV-GIC with message transmitter cooperation.*

5.3.2 Generalized Degrees of Freedom and Reciprocity

For the LV-GIC with message-only cooperation defined by $H = (h_{aa}, h_{ab}, h_{ba}, h_{bb})$, let the reciprocal channel be defined as $H'(h_{aa}, h_{ba}, h_{ab}, h_{bb})$, and consider the LV-GIC with receiver cooperation with gains given by H'. Furthermore, denote as the reciprocal view of each local view with message-only cooperation as a local view with receiver cooperation as defined in Table 5.1.

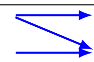
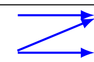
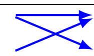
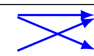
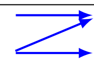
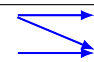
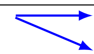
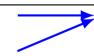
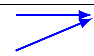
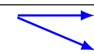
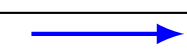
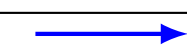
	View	Reciprocal	
1			3
2			2
3			1
4			6
5			5
6			4
7			7

Table 5.1: Reciprocal Views: The table depicts the local views that are reciprocal in the sense of opportunities enabled in receiver cooperation and transmitter cooperation (uplink and downlink) modes.

If one compares the outer bounds of the message-only cooperation scenario with the capacity region characterization of receiver cooperation LV-GIC, one will notice a striking similarity in the form of the regions. In fact the only difference in the regions results from a power gain in sum-rate of the message-only cooperation case, which stems from the possibility of correlated transmit signals. This effect of this power gain can be bounded, and provides us with the following result.

Corollary 34. *Let a view, V, and channel, H, be given and let V' and H' be the reciprocal view (as defined in Table 5.1) and reciprocal channel. The GDoF region of View V and channel H with message-only transmitter cooperation is contained with the GDoF region of View V' and channel H' with receiver cooperation.*

Similar reciprocities have been noted in other scenarios [49, 50, 8, 7, 52], and the possibility of such reciprocity is enticing in cellular systems where uplink and downlink use the same frequency resources; the uplink and downlink views and channel are likely to be reciprocal.

Whether there exist policies that achieve the outer bounds we have arrived at for the message-only transmitter cooperation case remains to be seen in general, and either affirmation or repudiation of the GDoF reciprocity would provide significant intuition regarding the impact of local view node knowledge.

5.4 Base Station Cooperation for Current Downlink Network Architecture

Downlink forms the majority of bits communicated in current cellular networks. In this section we consider what our results can say about inter-cell interference mitigation in current downlink architecture. In particular, we consider how the possible types of data permitted on the cooperative link can improve current systems.

Consider again two neighboring base stations, each of which has trained for channels to all users within its cell. In current systems, training across cells does not occur, and the resulting scenario is described by a LV-GIC with View 7. Recall that in §3.4 we showed that no protocol can outperform TDM for this scenario.

Now consider if the base stations share only their local views, but not messages. Transmitter b tells Transmitter a h_{bb} and Transmitter a tells Transmitter b h_{aa} and the resulting system is described by a LV-GIC with View 5. Again, we refer the reader to §3.4 where we showed that despite the added knowledge, still no protocol can significantly outperform TDM.

Similarly if only message data is shared, we showed in §5.3 that still no protocol can significantly outperform TDM. In fact, in order to exact any GDoF gain in the downlink scenario solely from cooperation, all knowledge at both links (both local view and user messages) must be shared. Moreover, the resulting gain still cannot be described as the same as the spatial gain that results from an ideal, full-knowledge cooperative transmission.

As we show in the next chapter, the approach of transmitter cooperation involving sharing of both views and message data compares favorably with the approach of each node dedicating more resources to learning the channel. When one considers the loss in wireless bandwidth needed to arrive at more complete local views, the cooperative approach becomes an attractive alternative in current cellular architecture.

5.5 Remarks

Remark 1: It remains to be seen if a DPC scheme may be adapted in the way we adapted superposition to jointly encode the weaker user's message in View 5. Because the successive DPC scheme wit fixed order generates a non-convex region, it is likely that in order to meet the local view MPC-satisfying capacity using DPC will require a more complex notion of time-sharing. between approaches.

Remark 2: The approach used in View 5 actually relies on the jointly encoded codewords not coherently combining (or canceling) in order to robustify the scheme to all of the possible value for unknown channel parameters. Consequently, there is some loss in SNR, however this gain can be bounded, and the pre-log factor is universally increased for all values of unknown channel parameter. To our knowledge, this is the first situation where such as scheme is required, and therefore the first time this scheme has been proposed.

The compound MIMO-BC, of which View 5 is a special example, have been studied previously [53], however the uncertainty set was significantly more general, and the authors focused on degrees of freedom (DoF) as the sole metric. We believe our special case provides a large bit of insight not only into the more general problem, but more importantly describes a realistic scenario in cooperative communications: cross-link gains are rarely measured in interference networks.

Remark 3: The capacity characterization for View 6 is tight mainly of the possibility of $h_{aa} = h_{ab}$, $h_{ba} = h_{bb}$. If the two receivers see the same exact (or phase rotated) channels, then the only option is to perform a schemes such as rate splitting or TDM.

Chapter 6

Conclusion

In this concluding chapter, we summarize our contributions, and suggest possible future directions.

6.1 Summary of Contributions

This dissertation is a first step towards better understanding how we may design wireless networks with distributed infrastructure that are able to effectively cope with interference. Our results on two-user local view Gaussian interference channels highlight the efficacy of each node learning specific subsets of channel gains, and how such gains may be matched or even superseded by incorporating cooperative elements between more powerful infrastructure nodes.

For completely non-cooperative interference networks, our results on the two-user interference channel suggest that

- For transmitters without knowledge of the direct link, the outgoing interference incurred upon another user, and at least one other channel gain, orthogonalization-based schemes such as TDM are optimal in that *no other scheme performs better*. This results justifies many approaches seen in practice, which either explicitly orthogonalize (TDM, FDM, frequency reuse, CDM, etc.) or stochastically attempt to orthogonalize (medium-access algorithms like ALOHA and 802.11) user transmissions.

- For transmitters with such knowledge — namely, those with either View 1 or View 2 from Figure 2.3— performing better than TDM is contingent on the local view revealing an opportunity for a node to increase rate.

With respect to cooperative networks, we studied cases where cooperation was one-sided, specifically modeling scenarios consistent with base station cooperation in cellular networks.

- In uplink communication, we considered sharing of channel output information and coherent decoding. We found that the presence of the cooperative link enables more opportunities for rate gain beyond TDM, and that such opportunities can occur in all but the most limited of local views (View 1). Uplink base station cooperation is also an attractive solution because we rely only on increased complexity at the base stations (and supporting core network), and mobile users need only have enough knowledge in the local view to capitalize on additional spatial opportunities.

- In downlink communication, we identified two types of knowledge that might be shared on the cooperative link: sharing local views and sharing messages. When sharing local views, we share knowledge about the network state while maintaining independence in encoding of messages. The first benefit of sharing local views is that nodes no longer have mismatched knowledge regarding channel state, and thus tighter coordination between selection of transmission parameters such as codebook structure and rate becomes possible. The second benefit is simply that transmitters have a more complete understanding of the interference structure of the network.

When sharing only messages, the network topology is transformed into one resembling a local view relay network: each transmitter is a source that must determine an end-to-end rate which may rely on uncertain assistance from the other transmitter. This scenario is inherently more complicated, and likely requires more complex coding schemes to achieve the outer bounds presented. One note of interest however, is the reciprocity between our presented GDoF outer bounds, and the GDoF region of a reciprocal local view interference channel with receiver cooperation.

Finally, we showed that relative to current cellular network downlink architecture (in which only the direct link is measured and very little knowledge is shared between base stations) performance gains beyond TDM require either sharing both local views and message data, or additional cross-cell channel training. Although our presentation of results is agnostic to the relative cost differences between cooperative sharing versus added channel training, we expect that exploiting the preexisting backhaul link between base stations poses an attractive option when contrasted with the additional cross-cell time and frequency synchronization, as well as time-frequency resources, required to enable additional training.

In addition to the intuitions gained about coping with interference, in the process of arriving at our results we developed a number of analytical tools and methodologies that either help to formalize the problem studied, or enable the derivation of new inner and outer bounds on capacity:

- Our notion of performance and capacity regions is qualified by the notion of a policy: a deterministic response to a local view. Policies are abstract models of physical and medium-access layer protocols, and we consider only policies which perform universally better than orthogonalization. In doing so, we establish a baseline level of performance regardless of channel state, which demonstrates in certain scenarios that one may not perform better than the baseline without incurring some corner case.

- We analyzed the local view interference channel by considering virtual Z-channels and unwrapping of the interference relationship between the two users. In doing so, we may establish a set of linear interferer-interferee relationships between different policy responses, and are better able to isolate limitations on performance.

- We developed a methodology for creating a genie-aided outer bound that approximates in the Gaussian interference domain the signal level-by-level intuitions that are explicit in the linear deterministic channel. This approach is especially relevant in local view analysis, since nodes may often be unsure of the strength of signals and thus how structures in the transmitted signals align.

- In considering message-only transmitter cooperation, we isolated one scenario where inability to coherently beamform results from incomplete local view, and thus a transmission technique that relies on both joint encoding and joint decoding of one user's message provides robustness against possible channel states. This specific scheme employs a single-user code at one transmitter (the transmitter with the weaker channel) and an independent superposition code at the other (stronger) transmitter. The weaker user's message is encoded at both transmitters so that regardless of how much interference is incurred, both messages may be decoded at their intended receivers.

The work presented in this dissertation may also be found with additional results and intuition in [30, 29].

6.2 Future Work

In this section we identify a number of both short and long term directions for extending our results.

Larger Networks

We have studied the smallest unit of any interference network. This is primarily justified by the clarity of the intuition gained from such an analysis, however we also note that in the application considered (inter-cell interference in cellular networks) the number of dominant interferers is likely to be low. The challenge in extending our results to large networks lies mainly in identifying topologies and local views of interest. Exhaustively considering all local views as we have done in this dissertation seems an especially intimidating affair, since the number of local views to consider would grow exponentially with the number of channel gains defining the network.

Two formalizations of "larger networks" however make sense in the cellular domain. In the first, one might consider K-user interference channels with a local view structure such that knowledge regarding all interferers is symmetric. For instance, if Transmitter a knows h_{ab} then it also knows h_{ac}, h_{ad}, h_{ae}, etc. Structuring the local view in such a way emulates the scenario in actual deployments where nodes treat all interferers identically, and thus their knowledge regarding specific parameters exhibits similarities across all interference sources. This analysis faces other challenges though, since the capacity region of the K-user IC is even more poorly characterized than the two-user case.

Another notion of large networks that would be interesting to consider is to consider a pair of interfering multiple-access channels or interfering broadcast. These two network topology would emulate the uplink and downlink communication modes of two adjacent cells where all users within both cells vie for usage of the same spectral resources.[1] Such topologies do not yet occur in current deployed systems, however it is possible to envision such a scenario becoming more and more prevalent as the number of mobile cells and users grows.

Extending the Local View Model

Our local view model for node knowledge is admittedly simple. In this work, we only consider the incompleteness and mismatched nature of knowledge, and ignore the notion of error in knowledge that results from noisy estimates. Furthermore, our model of incompleteness is on a parameter-by-parameter level: each channel gain was either perfectly known or completely unknown. While this models the act of learning versus not learning a specific channel, it does prevent us from considering cases such as coarse quantization of estimated channel gains or noisy channel estimates.

Fortunately, consideration of non-stochastic noise models and other forms of knowledge incompleteness (e.g., vector quantization of channel state), though potentially more complicated to analyze, fits well within our methodology. The particular generalization resulting from an assumed vector quantization of the network state parameter space, is particularly of interest, since such approaches are often used in practice and such a generalization also allows us to consider cases such as "we never have strong interference". The generalization of our results in such a manner is therefore of primary interest.

Further Applications

At its core, our local view model is a methodology for analysis of networks where nodes have incomplete and mismatched knowledge, which inhibits inter-node coordination. Our performance metric is similar to notions of robustness, and thus we envision that

[1] This is not the case in current 4G architecture, wherein intra-cell coordination of transmissions is accomplished through OFMDA, essentially orthogonalizing transmission.

the methodology used — e.g., modeling of knowledge incompleteness and mismatch and performance relative to a baseline policy — may be applied in networks other than wireless communication networks. In particular, networks where robustness is a primary concern (e.g., manufacturing processes, power grids, disease prevention and treatment) as well as networks where the statistical dynamics are either non-ergodic or poorly understood (e.g., social networks, financial markets) are possible domains for application of our methodology.

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Appendix A

Proofs

A.1 Local View MAC

Proof of Theorem 2. From the inequalities defining the boundary of the MAC capacity region, we have

$$\sum_{m=1}^{K} r_k\left(\widehat{G}_k\right)\Big|_{g_k=h} \leq h, \tag{A.1}$$

where h is any potential channel gain. Applying the minimum performance criterion yields

$$1 \leq \sum_{k=1}^{K} \frac{r_k\left(\widehat{G}_k\right)\Big|_{g_k=h}}{h} \leq 1, \tag{A.2}$$

or

$$\sum_{k=1}^{K} \frac{r_k\left(\widehat{G}_k\right)\Big|_{g_k=h}}{h} = 1. \tag{A.3}$$

Select K non-negative integer values, $h_1, \ldots, h_K$, and notice

$$\sum_{k=1}^{K} \frac{r_k\left(\widehat{G}_k\right)\Big|_{g_k=h_k}}{g_k} = \sum_{k=1}^{K} \frac{r_k\left(\widehat{G}_k\right)\Big|_{g_k=h_k}}{h_k} \tag{A.4}$$

$$= \sum_{k=1}^{K} \frac{r_k\left(\widehat{G}_k\right)\Big|_{g_k=h_k}}{h_k} + K - \sum_{\ell=1}^{K}\left(\sum_{k=1}^{K} \frac{r_k\left(\widehat{G}_k\right)\Big|_{g_k=h_\ell}}{h_\ell}\right) \tag{A.5}$$

$$= K - \sum_{\ell=1}^{K-1}\left(\left(\sum_{k=1}^{\ell-1} \frac{r_k\left(\widehat{G}_k\right)\Big|_{g_k=h_\ell}}{h_\ell}\right) + \frac{r_k\left(\widehat{G}_k\right)\Big|_{g_k=h_K}}{h_K} + \left(\sum_{k=\ell+1}^{K} \frac{r_k\left(\widehat{G}_k\right)\Big|_{g_k=h_\ell}}{h_\ell}\right)\right). \tag{A.6}$$

Applying the minimum performance criterion to the left hand side, as well as to each of the terms indexed by ℓ on the right hand side in (A.6), we have

$$1 \leq \sum_{k=1}^{K} \frac{r_k\left(\widehat{G}_k\right)\Big|_{g_k=h_k}}{g_k} \leq K - (K-1) = 1. \tag{A.7}$$

Since this holds for any non-negative values of $h_1, \ldots, h_K$ the theorem holds. □

A.2 Local View Linear Deterministic Interference Channels

LV-LDIC View 1

Proof of Theorem 3. First, we clarify that the capacity-achieving policy is specifically catered to the channel state considered, and may require the rate point achieved in other channel states to be on the TDM boundary.

Outer Bound

For any policy satisfying the minimum performance criteria, by definition there must exist non-negative τ_a and τ_b such that for all G,

$$\tau_a(g_{aa}, g_{bb}) + \tau_b(g_{aa}, g_{bb}) = 1, \tag{A.8}$$

and

$$r_a(\widehat{G}_a) \geq g_{aa}\tau_a(g_{aa}, g_{bb}), \tag{A.9}$$

$$r_b(\widehat{G}_b) \geq g_{bb}\tau_b(g_{aa}, g_{bb}), \tag{A.10}$$

where system-wide parameters are allowed to depend on the common knowledge (g_{aa} and g_{bb}).

Two virtual single Z-channels immediately result in bounds (3.106) and (3.108). At Receiver a, if $g_{ba} = g_{bb}$ we apply (3.57) and find

$$nr_a\left(\widehat{G}_a\right) \leq \left(ng_{ba} - \sum_{j=1}^{g_{ba}} L_{b,j}\left(\widehat{G}_b\right) + \sum_{i=1}^{g_{aa}-g_{ba}} L_{a,i}\left(\widehat{G}_a\right)\right)\Bigg|_{g_{ba}=g_{bb}} \tag{A.11}$$

$$\leq n\left(g_{bb} - r_b\left(\widehat{G}_b\right)\Big|_{g_{ba}=g_{bb}}\right) + \sum_{i=1}^{g_{aa}-g_{bb}} L_{a,i}\left(\widehat{G}_a\right) \tag{A.12}$$

$$\leq n\left[g_{aa} - \tau_b(g_{aa}, g_{bb})g_{bb}\right]. \tag{A.13}$$

Similarly, at Receiver b we have

$$nr_b\left(\widehat{G}_b\right) \leq ng_{aa}\tau_b(g_{aa}, g_{bb}). \tag{A.14}$$

To arrive at the other bounds, we note in (3.57) that regardless of the incoming interference gain, g_{ba}, the following statement is necessary for achievability:

$$nr_a(\widehat{G}_a) \leq \sum_{i=1}^{g_{ab}} L_{a,i}\left(\widehat{G}_a\right) + n\max(g_{aa} - g_{ab}, g_{ba}) - \sum_{j=1}^{g_{ba}} L_{b,j}\left(\widehat{G}_b\right). \tag{A.15}$$

In this expression, we draw a distinction between the entropy of the interference component and the entropy of the non-interfering component of the signal. To clarify analysis, we define the average entropies of the interference components of each transmitter's input as

$$\overline{r}_a^c\left(\widehat{G}_a\right) \triangleq \frac{1}{n}\sum_{i=1}^{g_{ab}} L_{a,i}\left(\widehat{G}_a\right), \tag{A.16}$$

$$\overline{r}_b^c\left(\widehat{G}_b\right) \triangleq \frac{1}{n}\sum_{j=1}^{g_{ba}} L_{b,j}\left(\widehat{G}_b\right). \tag{A.17}$$

Each transmitter's interference component and the non-interference component separately by constructing Z-channels both in the forward (adding virtual users that receive interference) and backward (adding virtual users that may induce interference) directions, as described in Section 3.1.2.1. Examples of the virtual Z-channels and their relation to specific bounds are shown in Figure A.1.

First, consider Transmitter a's interference component. If the Z-channel terminates at the next signal, the following are two conditions for achievability.

$$\overline{r}_a^c\left(\widehat{G}_a\right) \leq \max(g_{bb}, g_{ab}) - r_b\left(\widehat{G}_b'\right), \tag{A.18}$$

$$\overline{r}_a^c\left(\widehat{G}_a\right) \leq \frac{1}{n}\sum_{j=1}^{g_{ba}'} L_{b,j}\left(\widehat{G}_b'\right) + \max(g_{bb} - g_{ba}', g_{ab}) - r_b\left(\widehat{G}_b'\right) \tag{A.19}$$

$$= \overline{r}_b^c\left(\widehat{G}_b'\right) + \max(g_{bb} - g_{ba}', g_{ab}) - r_b\left(\widehat{G}_b'\right). \tag{A.20}$$

Similarly,

$$\overline{r}_b^c\left(\widehat{G}_b\right) \leq \max(g_{aa}, g_{ba}) - r_a\left(\widehat{G}'_a\right), \tag{A.21}$$

$$\overline{r}_b^c\left(\widehat{G}_b\right) \leq \overline{r}_a^c\left(\widehat{G}'_a\right) + \max(g_{aa} - g'_{ab}, g_{ba}) - r_a\left(\widehat{G}'_a\right). \tag{A.22}$$

Using expansions in the forward direction, we find the following four families of bounds where $M \in \mathbb{Z}^+$:

$$\begin{aligned}\overline{r}_a^c\left(\widehat{G}_a\right) \leq\ & \max(g_{bb} - g_{ba}^{(1)}, g_{ab}) - r_b\left(\widehat{G}_b^{(1)}\right) \\ & + \sum_{\theta=1}^{\Theta-2}\left[\max(g_{aa} - g_{ab}^{(\theta)}, g_{ba}^{(\theta)}) - \overline{r}_a\left(\widehat{G}_a^{(\theta)}\right) + \max(g_{bb} - g_{ba}^{(\theta+1)}, g_{ab}^{(\theta)}) - r_b\left(\widehat{G}_b^{(\theta+1)}\right)\right] \\ & + \max(g_{aa} - g_{ab}^{(\Theta-1)}, g_{ba}^{(\Theta-1)}) - \overline{r}_a\left(\widehat{G}_a^{(\Theta-1)}\right) + \max(g_{bb}, g_{ab}^{(\Theta-1)}) - r_b\left(\widehat{G}_b^{(\Theta)}\right),\end{aligned} \tag{A.23}$$

$$\begin{aligned}\overline{r}_a^c\left(\widehat{G}_a\right) \leq\ & \max(g_{bb} - g_{ba}^{(1)}, g_{ab}) - r_b\left(\widehat{G}_b^{(1)}\right) \\ & + \sum_{\theta=1}^{\Theta-2}\left[\max(g_{aa} - g_{ab}^{(\theta)}, g_{ba}^{(\theta)}) - \overline{r}_a\left(\widehat{G}_a^{(\theta)}\right) + \max(g_{bb} - g_{ba}^{(\theta+1)}, g_{ab}^{(\theta)}) - r_b\left(\widehat{G}_b^{(\theta+1)}\right)\right] \\ & + \max(g_{aa} - g_{ab}^{(M-1)}, g_{ba}^{(\Theta-1)}) - \overline{r}_a\left(\widehat{G}_a^{(\Theta-1)}\right) + \max(g_{bb} - g_{ba}^{(\Theta)}, g_{ab}^{(\Theta-1)}) \\ & - r_b\left(\widehat{G}_b^{(\Theta)}\right) + \overline{r}_b^c\left(\widehat{G}_b^{(\Theta)}\right),\end{aligned} \tag{A.24}$$

$$\begin{aligned}\overline{r}_a^c\left(\widehat{G}_a\right) \leq\ & \max(g_{bb} - g_{ba}^{(1)}, g_{ab}) - r_b\left(\widehat{G}_b^{(1)}\right) \\ & + \sum_{\theta=1}^{\Theta-2}\left[\max(g_{aa} - g_{ab}^{(\theta)}, g_{ba}^{(\theta)}) - \overline{r}_a\left(\widehat{G}_a^{(\theta)}\right) + \max(g_{bb} - g_{ba}^{(\theta+1)}, g_{ab}^{(\theta)}) - r_b\left(\widehat{G}_b^{(\theta+1)}\right)\right] \\ & + \max(g_{aa}, g_{ba}^{(\Theta)}) - \overline{r}_a\left(\widehat{G}_a^{(\Theta)}\right),\end{aligned} \tag{A.25}$$

$$\begin{aligned}\overline{r}_a^c\left(\widehat{G}_a\right) \leq\ & \max(g_{bb} - g_{ba}^{(1)}, g_{ab}) - r_b\left(\widehat{G}_b^{(1)}\right) \\ & + \sum_{\theta=1}^{\Theta-2}\left[\max(g_{aa} - g_{ab}^{(\theta)}, g_{ba}^{(\theta)}) - \overline{r}_a\left(\widehat{G}_a^{(\theta)}\right) + \max(g_{bb} - g_{ba}^{(\theta+1)}, g_{ab}^{(\theta)}) - r_b\left(\widehat{G}_b^{(\theta+1)}\right)\right] \\ & + \max(g_{aa} - g_{ab}^{(\Theta)}, g_{ba}^{(\Theta)}) - \overline{r}_a\left(\widehat{G}_a^{(\Theta)}\right) + \overline{r}_a^c\left(\widehat{G}_a^{(\Theta)}\right),\end{aligned} \tag{A.26}$$

which hold for any $\Theta \in \mathbb{Z}^+$, and arbitrary values of $G^{(\theta)}$. We tighten the bounds, by applying (A.9) and (A.10) and considering the values of Θ and $G^{(\theta)}$ that minimize the right hand sides of (A.23)–(A.26).

Applying (A.9) and (A.10), expression (A.23) becomes

$$\begin{aligned}\overline{r}_a^c\left(\widehat{G}_a\right) \leq\; & \max(g_{bb} - g_{ba}^{(1)}, g_{ab}) - g_{bb}\tau_b(g_{aa}, g_{bb}) \\ & + \sum_{\theta=1}^{\Theta-2}[\max(g_{aa} - g_{ab}^{(\theta)}, g_{ba}^{(\theta)}) + \max(g_{bb} - g_{ba}^{(\theta+1)}, g_{ab}^{(\theta)}) - (g_{bb} + \delta\tau_a(g_{aa}, g_{bb}))] \\ & + \max(g_{aa} - g_{ab}^{(\Theta-1)}, g_{ba}^{(\Theta-1)}) - g_{aa}\tau_a(g_{aa}, g_{bb}) + \max(g_{bb}, g_{ab}^{(\Theta-1)}) - g_{bb}\tau_b(g_{aa}, g_{bb})\end{aligned} \tag{A.27}$$

$$\begin{aligned}=\; & \max(g_{bb} - g_{ba}^{(1)}, g_{ab}) - (\Theta - 1)(g_{bb} + \delta\tau_a(g_{aa}, g_{bb})) - g_{bb}\tau_b(g_{aa}, g_{bb}) \\ & + \sum_{\theta=1}^{\Theta-2}[\max(g_{aa} - g_{ab}^{(\theta)}, g_{ba}^{(\theta)}) + \max(g_{bb} - g_{ba}^{(\theta+1)}, g_{ab}^{(\theta)})] \\ & + \max(g_{aa} - g_{ab}^{(\Theta-1)}, g_{ba}^{(\Theta-1)}) + \max(g_{bb}, g_{ab}^{(\Theta-1)}).\end{aligned} \tag{A.28}$$

In order to minimize this expression for a given Θ, we assign the values of $g_{ab}^{(\theta)}$ and $g_{ab}^{(\theta)}$ as

$$g_{ab}^{(\Theta-1)} = g_{bb}, \tag{A.29}$$

$$g_{ba}^{(\theta)} = g_{aa} - g_{ab}^{(\theta)}, \tag{A.30}$$

$$g_{ab}^{(\theta-1)} = \left(g_{bb} = g_{ba}^{(\theta)}\right)^+. \tag{A.31}$$

Substituting $\ell = \Theta - 1$ and noting $g_{ab} \geq 0$,we arrive at the first bound on interference component entropy

$$\overline{r}_a^c\left(\widehat{G}_a\right) \leq \max(g_{bb} - \ell\delta, g_{ab}) + \ell\delta\tau_b(g_{aa}, g_{bb}) - g_{bb}\tau_b(g_{aa}, g_{bb}). \tag{A.32}$$

We can also consider (A.23) and (A.25) where the terminal link represents the response to the optimized channel state (i.e., $g_{ba}^{(\Theta)} = g_{ba}$), and derive from (A.23) the bound

$$\overline{r}_a^c\left(\widehat{G}_a\right) + r_b\left(\widehat{G}_b\right) \leq \max(g_{bb} - \ell\delta, g_{ab}) + \ell\delta\tau_b(g_{aa}, g_{bb}). \tag{A.33}$$

For (A.24), selecting $g_{ba}^{(\Theta)} = g_{ba}$ yields

$$\begin{aligned}\overline{r}_a^c\left(\widehat{G}_a\right) + r_b\left(\widehat{G}_b\right) - \overline{r}_b^c\left(\widehat{G}_b\right) \leq \ & \max\left(g_{bb} - g_{ba}^{(1)}, g_{ab}\right) - (\Theta - 1)\left(g_{bb} + \delta\tau_a(g_{aa}, g_{bb})\right) \\ & + \sum_{\theta=1}^{\Theta-1}\left[\max\left(g_{aa} - g_{ab}^{(\theta)}, g_{ba}^{(\theta)}\right) + \max\left(g_{bb} - g_{ba}^{(\theta+1)}, g_{ab}^{(\theta)}\right)\right] \\ & + \max\left(g_{aa} - g_{ab}^{(\Theta-1)}, g_{ba}^{(\Theta-1)}\right) + \max\left(g_{bb} - g_{ba}, g_{ab}^{(\Theta-1)}\right).\end{aligned} \tag{A.34}$$

Selecting possible interference gains in the manner,

$$g_{ab}^{(\Theta-1)} = (g_{bb} - g_{ba})^+, \tag{A.35}$$

$$g_{ba}^{(\theta)} = g_{aa} - g_{ab}^{(\theta)}, \tag{A.36}$$

$$g_{ab}^{(\theta-1)} = (g_{bb} - g_{ba}^{(\theta)})^+, \tag{A.37}$$

results in the bound (given $\ell \geq 0$)

$$\overline{r}_a^c\left(\widehat{G}_a\right) + r_b\left(\widehat{G}_b\right) - \overline{r}_b^c\left(\widehat{G}_b\right) \leq \max(g_{bb} - g_{ba} - \ell\delta, g_{ab}) + \ell\delta\tau_b(g_{aa}, g_{bb}). \tag{A.38}$$

Similar analysis and choices for free parameters from the expressions (A.25) and

(A.26) yield the following bounds (given $\ell \geq 0$):

$$\overline{r}_a^c\left(\widehat{G}_a\right) \leq g_{ab} + \ell\delta\tau_b(g_{aa}, g_{bb}), \tag{A.39}$$

$$\overline{r}_a^c\left(\widehat{G}_a\right) + r_a\left(\widehat{G}_a\right) \leq g_{ab} + (\ell+1)\delta\tau_b(g_{aa}, g_{bb}) + g_{aa}\tau_a(g_{aa}, g_{bb}), \tag{A.40}$$

$$r_a\left(\widehat{G}_a\right) \leq \max(g_{aa}, g_{ab}) + \ell\delta\tau_b(g_{aa}, g_{bb}) - g_{bb}\tau_b(g_{aa}, g_{bb}). \tag{A.41}$$

Analogously, from expansion from the interference component of b, we have (given $\ell \geq 0$)

$$\overline{r}_b^c\left(\widehat{G}_b\right) \leq \max(g_{ba}, g_{aa}) - g_{aa}\tau_a(g_{aa}, g_{bb}) + \ell\delta\tau_b(g_{aa}, g_{bb}), \tag{A.42}$$

$$\overline{r}_b^c\left(\widehat{G}_b\right) + r_a\left(\widehat{G}_a\right) \leq \max(g_{ba}, g_{aa}) + \ell\delta\tau_b(g_{aa}, g_{bb}), \tag{A.43}$$

$$\begin{aligned}\overline{r}_b^c\left(\widehat{G}_a\right) + r_a\left(\widehat{G}_b\right) - \overline{r}_a^c\left(\widehat{G}_b\right) \leq{}& \max(g_{ba} - \ell\delta, (g_{aa} - g_{ab})^+, (g_{ba} - g_{ab})^+, (g_{ba} - g_{aa})^+) \\ &+ \ell\delta\tau_b(g_{aa}, g_{bb}),\end{aligned} \tag{A.44}$$

$$\overline{r}_b^c\left(\widehat{G}_b\right) \leq \max(g_{ba} - \ell\delta, (g_{ba} - g_{aa})^+) + \ell\delta\tau_b(g_{aa}, g_{bb}), \tag{A.45}$$

$$\begin{aligned}\overline{r}_b^c\left(\widehat{G}_a\right) + r_b\left(\widehat{G}_b\right) \leq{}& \max(g_{ba} - (\ell+1)\delta, (g_{ba} - g_{aa})^+) \\ &+ (g_{bb} + (\ell+1)\delta)\tau_b(g_{aa}, g_{bb}),\end{aligned} \tag{A.46}$$

$$r_b\left(\widehat{G}_b\right) \leq \max(g_{ba}, g_{aa}) - g_{aa}\tau_a(g_{aa}, g_{bb}) + \ell\delta\tau_b(g_{aa}, g_{bb}). \tag{A.47}$$

Although these bounds were derived by expanding a Z-channel forward from each transmitter's interference component, expressions (A.38) and (A.44) also account for the message component not contained in the interference signal: $r_b(\widehat{G}_b) - \overline{r}_b^c(\widehat{G}_b)$ and $r_a(\widehat{G}_a) - \overline{r}_a^c(\widehat{G}_a)$ respectively.

We now establish an additional pair of bounds for this component — what is representative of the 'private' message component in HK coding — derived from extension of a Z-channel in the reverse direction (Figure A.1(d)). To terminate each chain, we assume the interference gain of the final link is equal to the direct link gain.

Consequently, we have (given $\ell \geq 0$)

$$r_a\left(\widehat{G}_a\right) - \overline{r}_a^c\left(\widehat{G}_a\right) \leq \max((g_{aa} - g_{ab})^+, g_{bb} - \ell\delta) - g_{bb}\tau_b(g_{aa}, g_{bb}) + \ell\delta\tau_b(g_{aa}, g_{bb}), \tag{A.48}$$

$$r_a\left(\widehat{G}_a\right) - \overline{r}_a^c\left(\widehat{G}_a\right) \leq (g_{aa} - g_{ab})^+ + \ell\delta\tau_b(g_{aa}, g_{bb}), \tag{A.49}$$

$$r_b\left(\widehat{G}_b\right) - \overline{r}_b^c\left(\widehat{G}_b\right) \leq (g_{aa} + \ell\delta)\tau_b(g_{aa}, g_{bb}), \tag{A.50}$$

$$r_b\left(\widehat{G}_b\right) - \overline{r}_b^c\left(\widehat{G}_b\right) \leq (g_{bb} - g_{ba} - \ell\delta)^+ + \ell\delta\tau_b(g_{aa}, g_{bb}). \tag{A.51}$$

We remove redundant bounds from the signal component bounds derived thus far. For instance, (A.41), (A.47), and (A.50) are undeniably looser bounds than (A.13), (A.14), and (A.51) respectively. Additionally, (A.40) is the sum of (A.13) and (A.32). In addition to redundancies, the inequality (A.42) can be tightened by observing its relationship to (A.14): as a bound on a "public" component of the signal, the entropy bounded in (A.42) must be less than the entropies of the full transmitted signals.

$$\overline{r}_b^c\left(\widehat{G}_b\right) \leq g_{aa}\tau_b(g_{aa}, g_{bb}). \tag{A.52}$$

In summary, we have the following set of bounds

$$
\begin{aligned}
r_a\left(\widehat{G}_a\right) &\leq g_{aa} - g_{bb}\tau_b(g_{aa}, g_{bb}), && \text{(A.53)}\\
r_b\left(\widehat{G}_b\right) &\leq g_{aa}\tau_b(g_{aa}, g_{bb}), && \text{(A.54)}\\
\overline{r}_a^c\left(\widehat{G}_a\right) &\leq \min_{\ell\geq 0}[\max(g_{bb} - \ell\delta, g_{ab}) + \ell\delta\tau_b(g_{aa}, g_{bb})\\
&\quad - g_{bb}\tau_b(g_{aa}, g_{bb})], && \text{(A.55)}\\
\overline{r}_a^c\left(\widehat{G}_a\right) &\leq g_{ab}, && \text{(A.56)}\\
\overline{r}_b^c\left(\widehat{G}_b\right) &\leq g_{aa}\tau_b(g_{aa}, g_{bb}), && \text{(A.57)}\\
\overline{r}_b^c\left(\widehat{G}_b\right) &\leq \min_{\ell\geq 0}[\max(g_{ba} - \ell\delta, (g_{ba} - g_{aa})^+) + \ell\delta\tau_b(g_{aa}, g_{bb})], && \text{(A.58)}\\
\overline{r}_a^c\left(\widehat{G}_a\right) + r_b\left(\widehat{G}_b\right) &\leq \min_{\ell\geq 0}[\max(g_{bb} - \ell\delta, g_{ab}) + \ell\delta\tau_b(g_{aa}, g_{bb})], && \text{(A.59)}\\
\overline{r}_b^c\left(\widehat{G}_b\right) + r_a\left(\widehat{G}_a\right) &\leq \max(g_{ba}, g_{aa}), && \text{(A.60)}\\
\overline{r}_b^c\left(\widehat{G}_b\right) + r_b\left(\widehat{G}_b\right) &\leq \min_{\ell\geq 0}[\max(g_{ba} - (\ell+1)\delta, (g_{ba} - g_{aa})^+)\\
&\quad + (g_{aa} + \ell\delta)\tau_b(g_{aa}, g_{bb})], && \text{(A.61)}\\
\overline{r}_a^c\left(\widehat{G}_a\right) + r_b\left(\widehat{G}_b\right) - \overline{r}_b^c\left(\widehat{G}_b\right) &\leq \min_{\ell\geq 0}[\max(g_{ab}, g_{bb} - g_{ba} - \ell\delta) + \ell\delta\tau_b(g_{aa}, g_{bb})], && \text{(A.62)}\\
\overline{r}_b^c\left(\widehat{G}_b\right) + r_a\left(\widehat{G}_a\right) - \overline{r}_a^c\left(\widehat{G}_a\right) &\leq \min_{\ell\geq 0}[\max(g_{ba} - \ell\delta, (g_{aa} - g_{ab})^+, g_{ba} - g_{ab}, g_{ba} - g_{aa})\\
&\quad + \ell\delta\tau_b(g_{aa}, g_{bb})], && \text{(A.63)}\\
r_a\left(\widehat{G}_a\right) - \overline{r}_a^c\left(\widehat{G}_a\right) &\leq \min_{\ell\geq 0}[\max((g_{aa} - g_{ab})^+, g_{bb} - \ell\delta) - g_{bb}\tau_b(g_{aa}, g_{bb})\\
&\quad + \ell\delta\tau_b(g_{aa}, g_{bb})], && \text{(A.64)}\\
r_a\left(\widehat{G}_a\right) - \overline{r}_a^c\left(\widehat{G}_a\right) &\leq (g_{aa} - g_{ab})^+, && \text{(A.65)}\\
r_b\left(\widehat{G}_b\right) - \overline{r}_b^c\left(\widehat{G}_b\right) &\leq \min_{\ell\geq 0}[(g_{bb} - g_{ba} - \ell\delta)^+ + \ell\delta\tau_b(g_{aa}, g_{bb})]. && \text{(A.66)}
\end{aligned}
$$

Achievable Scheme

To complete the proof we have three tasks:

1. Define a policy that specifies the transmission scheme not only for the channel state at hand, but for all states with the same direct link gains

2. Show that any rate point on the outer bound can be achieved by such a scheme

3. Confirm that the rates prescribed for other channel states are achievable and satisfy the minimum specified rate.

As in View 2 and [19], the scheme used relies on each message being divided into a common component and a private component. Generally speaking, for each channel state, the common component of Sender a's codebook is generated by randomly selecting nr_a^c codewords from the set of all $n \times \max(g_{ab}, g_{aa})$ binary matrices. The private message codebook is generated by randomly selecting nr_a^p codewords from the set of $n \times (g_{aa} - g_{ab})^+$ matrices. On outgoing links that interfere with Link b, the g_{ab} most significant levels of the common message are sent. If $g_{aa} - g_{ab} > 0$, then the modulo addition of the private message and lower levels of the common message is transmitted. Similarly for Sender b, r_b^c and r_b^p govern the number of codewords (randomly drawn) in the common and private codebooks of Link b.

The size of component codebooks varies for different channel states, and is a function of each sender's local view. For the channel state being considered, the local views of the channel state G are $\widehat{G}_a$ and $\widehat{G}_b$, and the number of codewords in each component codebook are chosen such that $r_a^c(\widehat{G}_a)$, $r_a^p(\widehat{G}_a)$, $r_a(\widehat{G}_a)$, $r_b^c(\widehat{G}_b)$,$r_b^p(\widehat{G}_b)$, and $r_b(\widehat{G}_b)$ obey (3.106)–(3.119).

We assume joint decoding of all received components — each receiver perceives a virtual three-user MAC — which implies that the proposed policy is achievable for channel state if

$$r_a^c \leq g_{ab}, \tag{A.67}$$

$$r_a^p \leq (g_{aa} - g_{ab})^+, \tag{A.68}$$

$$r_a^c + r_a^p \leq g_{aa}, \tag{A.69}$$

$$r_b^c \leq g_{ba}, \tag{A.70}$$

$$r_b^p \leq (g_{bb} - g_{ba})^+, \tag{A.71}$$

$$r_b^c + r_b^p \leq g_{bb}, \tag{A.72}$$

$$r_a^p + r_b^c \leq \max(g_ba, g_{aa} - g_{ab}), \tag{A.73}$$

$$r_a^c + r_a^p + r_b^c \leq \max(g_ba, g_{aa}), \tag{A.74}$$

$$r_b^p + r_a^c \leq \max(g_ab, g_{bb} - g_{ba}), \tag{A.75}$$

$$r_a^c + r_a^p + r_b^c \leq \max(g_ab, g_{bb}). \tag{A.76}$$

Noting that the restrictions imposed in (3.106)–(3.119) are actually stricter than (A.67)–(A.76), the policy proposed thus far is achievable for the channel state considered completing Step 2.

For the responses to other channel states with local views $\widehat{G}'_a \neq \widehat{G}_a$ and $\widehat{G}'_b \neq \widehat{G}_b$, the public and private codebook sizes (rates) must also conform to a similar set of bounds such that the remain consistent with the responses of the considered channel state. Applying a similar virtual Z-channel expansion to arbitrary channel states, and assuming

$$r_a(\widehat{G}'_a) = r_a^c(\widehat{G}'_a) + r_a^p(\widehat{G}'_a) \geq g_{aa}\tau_a(g_{aa}, g_{bb}), \tag{A.77}$$

$$r_b(\widehat{G}'_b) = r_b^c(\widehat{G}'_b) + r_b^p(\widehat{G}'_b) \geq g_{bb}\tau_b(g_{aa}, g_{bb}), \tag{A.78}$$

we find for local views $\widehat{G}''_a \neq \widehat{G}_a$ and $\widehat{G}''_b \neq \widehat{G}_b$ and $\ell \geq 0$

$$r^c_a(\widehat{G}'_a) \leq g'_{ab}, \tag{A.79}$$

$$r^c_a(\widehat{G}'_a) \leq \max(g'_{ab}, g_{bb} - g_{ba} - \ell\delta) + \ell\delta\tau_b(g_{aa}, g_{bb}) - r^p_b(\widehat{G}_b), \tag{A.80}$$

$$\begin{aligned} r^c_a(\widehat{G}'_a) \leq\ & \max(g'_{ab}, g_{bb} + g_{ab} - g_{aa} - \ell\delta) + \ell\delta\tau_b(g_{aa}, g_{bb}) + (g_{aa} - g_{ab})^+ \\ & - g_{bb}\tau_b(g_{aa}, g_{bb}) - r^p_a(\widehat{G}_a), \end{aligned} \tag{A.81}$$

$$r^c_a(\widehat{G}'_a) \leq \max(g'_{ab}, g_{bb} - \ell\delta) + \ell\delta\tau_b(g_{aa}, g_{bb}) - r_b(\widehat{G}_b), \tag{A.82}$$

$$\begin{aligned} r^c_a(\widehat{G}'_a) \leq\ & \max(g'_{ab}, g_{bb} - \ell\delta) + \ell\delta\tau_b(g_{aa}, g_{bb}) + g_{aa} \\ & - g_{bb}\tau_b(g_{aa}, g_{bb}) - r_a(\widehat{G}_a), \end{aligned} \tag{A.83}$$

$$r^p_a(\widehat{G}'_a) \leq (g_{aa} - g'_{ab})^+, \tag{A.84}$$

$$r^p_a(\widehat{G}'_a) \leq \max(g_{ba} - \ell\delta, g_{aa} - g'_{ab}) + \ell\delta\tau_b(g_{aa}, g_{bb}) - r^c_b(\widehat{G}_b), \tag{A.85}$$

$$\begin{aligned} r^p_a(\widehat{G}'_a) \leq\ & (g_{aa} - g'_{ab})^+ - g_{bb}\tau_b(g_{aa}, g_{bb}) + \max(g_{ab}, g_{bb} - (g_{aa} - g'_{ab}) - \ell\delta) \\ & + \ell\delta\tau_b(g_{aa}, g_{bb}) - r^c_a(\widehat{G}_a), \end{aligned} \tag{A.86}$$

$$r^p_a(\widehat{G}'_a) + r^c_a(\widehat{G}'_a) \leq g_{aa}, \tag{A.87}$$

$$r^p_a(\widehat{G}'_a) + r^c_a(\widehat{G}'_a) \leq \max(g_{ba} - \ell\delta, g_{aa}) + \ell\delta\tau_b(g_{aa}, g_{bb}) - r^c_b(\widehat{G}_b), \tag{A.88}$$

$$r^p_a(\widehat{G}'_a) + r^c_a(\widehat{G}'_a) \leq cg_{ab} + g_{aa} - r^c_a(\widehat{G}_a) - g_{bb}\tau_b(g_{aa}, g_{bb}), \tag{A.89}$$

$$r^c_b(\widehat{G}'_a) \leq g'_{ba}, \tag{A.90}$$

$$r^c_b(\widehat{G}'_a) \leq \max(g'_{ba} - \ell\delta, g_{aa} - g_{ab}) + \ell\delta\tau_b(g_{aa}, g_{bb}) - r^p_a(\widehat{G}_a), \tag{A.91}$$

$$\begin{aligned} r^c_b(\widehat{G}'_a) \leq\ & \max(g'_{ba} - \ell\delta, g_{aa} - (g_{bb} - g_{ba})) + \ell\delta\tau_b(g_{aa}, g_{bb}) \\ & - g_{aa}\tau_a(g_{aa}, g_{bb}) + (g_{bb} - g_{ba})^+ - r^p_b(\widehat{G}_b), \end{aligned} \tag{A.92}$$

$$r^c_b(\widehat{G}'_a) \leq \max(g'_{ba} - \ell\delta, g_{aa}) + \ell\delta\tau_b(g_{aa}, g_{bb}) - r_a(\widehat{G}_a), \tag{A.93}$$

$$\begin{aligned} r^c_b(\widehat{G}'_a) \leq\ & \max(g'_{ba} - \ell\delta, \delta) + \ell\delta\tau_b(g_{aa}, g_{bb}) - g_{aa}\tau_a(g_{aa}, g_{bb}) \\ & + g_{bb} - r_b(\widehat{G}_b), \end{aligned} \tag{A.94}$$

$$r_b^p(\widehat{G}_a') \leq \left(g_{bb} - g_{ba}'\right)^+, \tag{A.95}$$

$$r_b^p(\widehat{G}_b') \leq \max(g_{ab}, g_{bb} - g_{ba}' - \ell\delta) + \ell\delta\tau_b(g_{aa}, g_{bb}) - r_a^c(\widehat{G}_a), \tag{A.96}$$

$$r_b^p(\widehat{G}_b') \leq \max(g_{aa} - g_{ba}, g_{bb} - g_{ba}') - g_{aa}\tau_a(g_{aa}, g_{bb}) + g_{ba} - r_b^c(\widehat{G}_b), \tag{A.97}$$

$$r_b^p(\widehat{G}_b') + r_b^c(\widehat{G}_b') \leq g_{bb}, \tag{A.98}$$

$$r_b^p(\widehat{G}_b') + r_b^c(\widehat{G}_b') \leq \max(g_{ab}, g_{bb} - \ell\delta) + \ell\delta\tau_b(g_{aa}, g_{bb}) - r_a^c(\widehat{G}_a), \tag{A.99}$$

$$\begin{aligned} r_b^p(\widehat{G}_b') + r_b^c(\widehat{G}_b') \leq\ & \max(g_{ba} + g_{bb} - \ell\delta, g_{aa}) + \ell\delta\tau_b(g_{aa}, g_{bb}) - r_b^c(\widehat{G}_b) \\ & - g_{aa}\tau_a(g_{aa}, g_{bb}). \end{aligned} \tag{A.100}$$

Using these expressions along with (3.106)–(3.119), we define the rates (and by proxy size) of codebooks in each policy response. Moreover, substitution of (3.106)–(3.119) into (A.79)–(A.100) we see that the rate satisfies the TDM criterion as desired. □

LV-LDIC View 2

Proof of Theorem 4.

Outer Bound

We first consider two limiting cases. If $g_{aa} = g_{bb} = g_{ab}$, by considering (3.45), we have

$$\begin{aligned} r_a\left(\widehat{G}_a\right)\Big|_{g_{aa}=g_{ab}} + r_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{ab}} &\leq g_{ab} \\ &= g_{ab}, \end{aligned} \tag{A.101, A.102}$$

where the equality is enforced in order to satisfy the minimum performance criterion. Similarly, if $g_{aa} = g_{bb} = g_{ba}$, we have

$$r_a\left(\widehat{G}_a\right)\Big|_{g_{aa}=g_{ba}} + r_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{ba}} = g_{ba}. \tag{A.103}$$

These two cases can be restated as

$$\frac{r_a\left(\widehat{G}_a\right)\Big|_{g_{aa}=g_{ab}}}{g_{ab}}+\frac{r_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{ab}}}{g_{ab}}=1, \tag{A.104}$$

$$\frac{r_a\left(\widehat{G}_a\right)\Big|_{g_{aa}=g_{ba}}}{g_{ba}}+\frac{r_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{ba}}}{g_{ba}}=1. \tag{A.105}$$

and when summed we have

$$\frac{r_a\left(\widehat{G}_a\right)\Big|_{g_{aa}=g_{ab}}}{g_{ab}}+\frac{r_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{ba}}}{g_{ba}}=2-\frac{r_a\left(\widehat{G}_a\right)\Big|_{g_{aa}=g_{ba}}}{g_{ba}}-\frac{r_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{ab}}}{g_{ab}}\leq 1, \tag{A.106}$$

where (A.106) is due to the TDM constraint. By applying the same constraint on the other side, we have

$$1\leq\frac{r_a\left(\widehat{G}_a\right)\Big|_{g_{aa}=g_{ab}}}{g_{ab}}+\frac{r_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{ba}}}{g_{ba}}\leq 1, \tag{A.107}$$

which implies that the two cases discussed are not only both constrained to a region where TDM is sufficient, but the operating points must be consistent, i.e.,

$$\frac{r_a\left(\widehat{G}_a\right)\Big|_{g_{aa}=g_{ab}}}{g_{ab}}=\frac{r_a\left(\widehat{G}_a\right)\Big|_{g_{aa}=g_{ba}}}{g_{ba}}=\tau_a\left(g_{ab},g_{ba}\right), \tag{A.108}$$

$$\frac{r_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{ab}}}{g_{ab}}=\frac{r_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{ba}}}{g_{ba}}=\tau_b\left(g_{ab},g_{ba}\right), \tag{A.109}$$

$$\tau_a\left(g_{ab},g_{ba}\right)+\tau_b\left(g_{ab},g_{ba}\right)=1. \tag{A.110}$$

For other cases of direct link gain, we assume the viewpoint of Transmitter a in considering its policy options. As we have shown, there must exist $\tau_a(g_{ab},g_{ba})$ and

$\tau_a(g_{ab}, g_{ba})$ summing to one, such that

$$r_a\left(\widehat{G}_a\right) \geq g_{aa}\tau_a\left(g_{ab}, g_{ba}\right), \tag{A.111}$$

$$r_b\left(\widehat{G}_b\right) \geq g_{bb}\tau_b\left(g_{ab}, g_{ba}\right). \tag{A.112}$$

When $g_{aa} \notin \{g_{ab}, g_{ba}\}$, there still exists a possibility of the other direct link being fully interfering/interfered, $g_{bb} \in \{g_{ab}, g_{ba}\}$. Therefore, regardless of the known direct link, the channel input and decoding process must both accommodate the constraints imposed by these two limiting possibilities, resulting in the virtual Z-channel shown in Figure A.2 for Transmitter a's view.

This provides us with three constraints related to the limiting cases, and one which is essentially the point-to-point capacity:

$$r_a\left(\widehat{G}_a\right) \leq \frac{1}{n}\sum_{i=1}^{g_{aa}} L_{a,i}\left(\widehat{G}_a\right) \tag{A.113}$$

$$\leq g_{aa}. \tag{A.114}$$

Of the remaining three bounds, we begin with the first with one resulting from an adaptation of (2.14) and isolating the lower of the two Z-channels depicted in the three-user Z-channel,

$$\sum_{i=1}^{g_{ab}} L_{a,i}\left(\widehat{G}_a\right) \leq \sum_{j=1}^{g_{bb}-g_{ab}} L_{b,j}\left(\widehat{G}_b\right) + ng_{ab} - nr_b(\widehat{G}_b) \tag{A.115}$$

$$\leq ng_{ab} - nr_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{ab}} \tag{A.116}$$

$$\leq ng_{ab}\tau_a\left(g_{ab}, g_{ba}\right), \tag{A.117}$$

which we apply in

$$r_a\left(\widehat{G}_a\right) \leq \frac{1}{n}\sum_{i=1}^{g_{aa}} L_{a,i}\left(\widehat{G}_a\right) \tag{A.118}$$

$$= \frac{1}{n}\left(\sum_{i=1}^{g_{ab}} L_{a,i}\left(\widehat{G}_a\right) + \sum_{i=g_{ab}+1}^{g_{aa}} L_{a,i}\left(\widehat{G}_a\right)\right) \tag{A.119}$$

$$\leq g_{ab}\tau_a\left(g_{ab}, g_{ba}\right) + \left(g_{aa} - g_{ab}\right)^{+}. \tag{A.120}$$

For the third bound on $r_a(\widehat{G}_a)$, we recall (2.13) and isolate only the upper of the two Z-channels:

$$r_a\left(\widehat{G}_a\right) \leq \frac{1}{n}\left(\sum_{i=1}^{g_{aa}-g_{ba}} L_{a,i}\left(\widehat{G}_a\right) - \sum_{j=1}^{g_{ba}} L_{b,j}\left(\widehat{G}_b\right)\right) + g_{ba} \tag{A.121}$$

$$\leq \frac{1}{n}\left(\sum_{i=1}^{g_{aa}-g_{ba}} l_{a,i}\left(\widehat{G}_a\right) + ng_{ba} - \sum_{j=1}^{g_{ba}} L_{b,j}\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{ba}}\right) \tag{A.122}$$

$$\leq \frac{1}{n}\left(\sum_{i=1}^{g_{aa}-g_{ba}} L_{a,i}\left(\widehat{G}_a\right) + ng_{ba} - nr_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{ba}}\right) \tag{A.123}$$

$$= \frac{1}{n}\left(\sum_{i=1}^{g_{aa}-g_{ba}} L_{a,i} + ng_{ba} - ng_{ba}\tau_b\left(g_{ab}, g_{ba}\right)\right) \tag{A.124}$$

$$\leq \left(g_{aa} - g_{ba}\right)^{+} + g_{ba}\tau_a\left(g_{ab}, g_{ba}\right). \tag{A.125}$$

For the fourth bound we also apply (2.13) but deviate at (A.124):

$$r_a\left(\widehat{G}_a\right) \leq \frac{1}{n}\left(\sum_{i=1}^{g_{aa}-g_{ba}} L_{a,i} + ng_{ba} - ng_{ba}\tau_b\left(g_{ab}, g_{ba}\right)\right) \tag{A.126}$$

$$\leq \frac{1}{n}\sum_{i=1}^{g_{aa}-g_{ba}} L_{a,i}\left(\widehat{G}_a\right) + g_{ba}\tau_a\left(g_{ab}, g_{ba}\right) \tag{A.127}$$

$$\leq \frac{1}{n}\sum_{i=1}^{g_{ab}} L_{a,i} + \frac{1}{n}\sum_{i=g_{ab}+1}^{g_{aa}-g_{ba}} L_{a,i} + g_{ba}\tau_a(g_{ab}, g_{ba}) \tag{A.128}$$

$$\leq g_{ab}\tau_a\left(g_{ab}, g_{ba}\right) + \left(g_{aa} - g_{ab} - g_{ba}\right)^{+} + g_{ba}\tau_a\left(g_{ab}, g_{ba}\right). \tag{A.129}$$

In summary, we have for User a:

$$r_a\left(\widehat{G}_a\right) \leq g_{aa}, \tag{A.130}$$

$$r_a\left(\widehat{G}_a\right) \leq \left(g_{aa} - g_{ab}\right)^+ + g_{ab}\tau_a\left(g_{ab}, g_{ba}\right), \tag{A.131}$$

$$r_a\left(\widehat{G}_a\right) \leq \left(g_{aa} - g_{ba}\right)^+ + g_{ba}\tau_a\left(g_{ab}, g_{ba}\right), \tag{A.132}$$

$$r_a\left(\widehat{G}_a\right) \leq \left(g_{aa} - g_{ab} - g_{ba}\right)^+ + \left(g_{ab} + g_{ba}\right)\tau_a\left(g_{ab}, g_{ba}\right). \tag{A.133}$$

The analogous bounds on the rate chosen by Sender b follow the same process.

Achievable Scheme

The scheme used in this scenario is the deterministic model version of the simple Han-Kobayashi scheme proposed in [19], and described in Section 3.1.1.2.

We generate public and private codebooks using random codes. For the public message of Sender a, let the $\nu_a = \min(g_{aa}, g_{ab})$. We choose $n\nu_a r_a^c(\widehat{G}_a)$ codewords randomly from the set of all $n \times \nu_a$ binary vectors using a uniform distribution over the set. If $g_{ab} < g_{aa}$ we also choose $n\nu_a r_a^p(\widehat{G}_a)$ codewords randomly from the set of all $n \times (g_{aa} - g_{ab})$ binary vectors again using a uniform distribution. At Sender b we do the same for $n\nu_b = \min(g_{bb}, g_{ba})$, $r_b^c(\widehat{G}_b)$ and $r_b^c(\widehat{G}_b)$.

From (3.30)–(3.41), the set of decodable rates $r_a^c(\widehat{G}_a)$, $r_a^p(\widehat{G}_a)$, and $r_b^c(\widehat{G}_b)$ at Receiver a is given by

$$r_a^c\left(\widehat{G}_a\right) \leq \min(g_{aa}, g_{ab}), \tag{A.134}$$

$$r_a^p\left(\widehat{G}_a\right) \leq (g_{aa} - g_{ab})^+, \tag{A.135}$$

$$r_b^c\left(\widehat{G}_b\right) \leq \min(g_{bb}, g_{ba}), \tag{A.136}$$

$$r_a^c\left(\widehat{G}_a\right) + r_a^p\left(\widehat{G}_a\right) \leq g_{aa}, \tag{A.137}$$

$$r_a^c\left(\widehat{G}_a\right) + r_b^c\left(\widehat{G}_b\right) \leq \max(g_{aa}, g_{ba}), \tag{A.138}$$

$$r_a^p\left(\widehat{G}_a\right) + r_b^c\left(\widehat{G}_b\right) \leq \max(g_{aa} - g_{ab}, g_{ba}), \tag{A.139}$$

$$r_a^c\left(\widehat{G}_a\right) + r_a^p\left(\widehat{G}_a\right) + r_b^c\left(\widehat{G}_b\right) \leq \max(g_{aa}, g_{ba}). \tag{A.140}$$

Since it is necessary for Sender a to know the rate of Sender b's public message in order to determine limits on its own public and private rates, we impose the constraints

$$r_a^c\left(\widehat{G}_a\right) \leq g_{ab}\tau_a\left(g_{ab}, g_{ba}\right), \tag{A.141}$$

$$r_b^c\left(\widehat{G}_b\right) \leq g_{ba}\tau_b\left(g_{ab}, g_{ba}\right), \tag{A.142}$$

chosen based on our understanding of the two limiting cases in the outer bound. Furthermore, we note that in order to satisfy the TDM constraint

$$r_a^c\left(\widehat{G}_a\right)|_{g_{aa}=g_{ab}} = r_a\left(\widehat{G}_a\right)|_{g_{aa}=g_{ab}} = g_{ab}\tau_a\left(g_{ab}, g_{ba}\right), \tag{A.143}$$

$$r_b^c\left(\widehat{G}_b\right)|_{g_{bb}=g_{ba}} = r_b\left(\widehat{G}_a\right)|_{g_{bb}=g_{ba}} = g_{ba}\tau_b\left(g_{ab}, g_{ba}\right). \tag{A.144}$$

The resulting region of rates achievable for the public and private messages of a

are

$$
\begin{aligned}
r_a^c\left(\widehat{G}_a\right) &\leq \min\left(g_{aa}, g_{ab}\right), && \text{(A.145)}\\
r_a^p\left(\widehat{G}_a\right) &\leq \left(g_{aa} - g_{ab}\right)^+, && \text{(A.146)}\\
r_a^c\left(\widehat{G}_a\right) + r_a^p\left(\widehat{G}_a\right) &\leq g_{aa}, && \text{(A.147)}\\
r_a^c\left(\widehat{G}_a\right) &\leq \max\left(g_{aa}, g_{ba}\right) - r_b^c\left(\widehat{G}_b\right) && \\
&\leq \max\left(g_{aa}, g_{ba}\right) - g_{ba}\tau_b\left(g_{ab}, g_{ba}\right) && \text{(A.148)}\\
&= \left(g_{aa} - g_{ba}\right)^+ + g_{ba}\tau_a\left(g_{ab}, g_{ba}\right), && \text{(A.149)}\\
r_a^p\left(\widehat{G}_a\right) &\leq \max\left(g_{aa} - g_{ab}, g_{ba}\right) - r_b^c\left(\widehat{G}_b\right) && \text{(A.150)}\\
&\leq \max\left(g_{aa} - g_{ab}, g_{ba}\right) - g_{ba}\tau_b\left(g_{ab}, g_{ba}\right) && \text{(A.151)}\\
&= \left(g_{aa} - g_{ab} - g_{ba}\right)^+ + g_{ba}\tau_a\left(g_{ab}, g_{ba}\right), && \text{(A.152)}\\
r_a^c\left(\widehat{G}_a\right) + r_a^p\left(\widehat{G}_a\right) &\leq \max\left(g_{aa}, g_{ba}\right) - r_b^c\left(\widehat{G}_b\right) && \text{(A.153)}\\
&\leq \max\left(g_{aa}, g_{ba}\right) - g_{ba}\tau_b\left(g_{ab}, g_{ba}\right) && \text{(A.154)}\\
&= \left(g_{aa} - g_{ba}\right)^+ + g_{ba}\tau_a\left(g_{ab}, g_{ba}\right). && \text{(A.155)}
\end{aligned}
$$

Which when simplified under the assumption $r_a(\widehat{G}_a) = r_a^c(\widehat{G}_a) + r_a^p(\widehat{G}_a)$, corresponds with the outer bounds of (A.130)–(A.133). Similar analysis of Sender b's scheme yields the analogous result. □

LV-LDIC Views 3 & 5

Proof of Theorem 6. As in View 1, by definition of the problem and noting the knowledge common to both transmitters, the following must hold for some $\tau_a(g_{aa}, g_{bb})$,

$\tau_b(g_{aa}, g_{bb})$ summing to one.

$$r_a(\widehat{G}_a) \geq g_{aa}\tau_a(g_{aa}, g_{bb}), \tag{A.156}$$

$$r_b(\widehat{G}_b) \geq g_{bb}\tau_b(g_{aa}, g_{bb}). \tag{A.157}$$

Our proof relies upon consideration of virtual Z-channels that provide structure to the uncertainty of each transmitter.

Let us first consider the POV of Transmitter a. Recalling the assumption that $g_{aa} \geq g_{bb}$, we consider all possible weak interference gain values for Transmitter a's out-going interference: $g_{ab} \in \{0, 1, \ldots, g_{aa}\}$. At Receiver b, the achievability of desired rates $r_b(\widehat{G}_b)$ is dependent on the following conditions

$$r_b\left(\widehat{G}_b\right)\Big|_{g_{ab}=0} \leq \frac{1}{n}\sum_{j=1}^{g_{bb}} L_{b,j}\left(\widehat{G}_b\right)\Big|_{g_{ab}=0}, \tag{A.158}$$

$$r_b\left(\widehat{G}_b\right)\Big|_{g_{ab}=1} \leq \frac{1}{n}\left(\sum_{j=1}^{g_{bb}-1} L_{b,j}\left(\widehat{G}_b\right)\Big|_{g_{ba}=1} + n - L_{a,1}\left(\widehat{G}_a\right)\right), \tag{A.159}$$

$$\vdots$$

$$r_b\left(\widehat{G}_b\right)\Big|_{g_{ab}=g_{aa}-1} \leq \frac{1}{n}\left(ng_{bb} - \sum_{i=g_{aa}-g_{bb}}^{g_{aa}-1} L_{a,i}\left(\widehat{G}_a\right)\right), \tag{A.160}$$

$$r_b\left(\widehat{G}_b\right)\Big|_{g_{ab}=g_{aa}} \leq \frac{1}{n}\left(ng_{bb} - \sum_{i=g_{aa}-g_{bb}+1}^{g_{aa}-1} L_{a,i}\left(\widehat{G}_a\right)\right). \tag{A.161}$$

Combining (A.158)–(A.161) with expression (A.157) implies more generally

$$\sum_{i=\kappa+1}^{\kappa+g_{bb}} L_{a,i}\left(\widehat{G}_a\right) \leq n\left(g_{bb} - g_{bb}\tau_b(g_{aa}, g_{bb})\right) \tag{A.162}$$

$$= n\left(g_{bb}\tau_a(g_{aa}, g_{bb})\right), \tag{A.163}$$

for $\kappa \in \{0, \ldots, g_{aa} - g_{bb}\}$; i.e. any g_{bb} successive layers of Transmitter a's input are constrained to a TDM-like rate. Notice that if g_{aa} is a multiple of g_{bb}, we can select disjoint sets of successive signal layers that span Transmitter a's input, thus completing the proof.

On the other hand, if g_{aa} is not evenly divisible by g_{bb}, we construct a virtual Z-channel with the actual Transmitter a as the initial interferer (the top link in Figure A.3(b)). Notice that in doing so, we neglect the incoming interference link gain g_{ab}, however this can be rationalized as a genie providing the interference signal to Receiver a. Moreover, we will demonstrate that it is in fact the objective of not inhibiting the transmission of the other link that proves the active constraint on Transmitter a's input.

Let $\theta_0 = g_{aa} \mod g_{bb}$, and notice that $\theta_0 < g_{bb} \leq g_{aa}$ — in Figure A.3(b), $\theta_0 = 1$. By properly selecting inequalities of the form (A.163) and also including the bound from (A.158)–(A.161) for $r_b(\widehat{G}_b)|_{g_{ab}=\theta_0}$, we have

$$nr_a(\widehat{G}_a) \leq \sum_{i=1}^{g_{aa}} L_{a,i}\left(\widehat{G}_a\right) \tag{A.164}$$

$$\leq n\left(g_{aa}-\theta_0\right)\tau_a\left(g_{aa},g_{bb}\right) + \sum_{i=1}^{\theta_0} L_{a,i}\left(\widehat{G}_a\right) \tag{A.165}$$

$$\leq n\left(g_{aa}-\theta_0\right)\tau_a\left(g_{aa},g_{bb}\right) + \sum_{j=1}^{g_{bb}-\theta_0} L_{b,j}\left(\widehat{G}_b\right)\Big|_{g_{ab}=\theta_0} + n\theta_0 - ng_{bb}\tau_b\left(g_{aa},g_{bb}\right). \tag{A.166}$$

In our virtual Z-channel, we now have a virtual b link where $g_{ba} = \theta_0$. We must now consider the constraints on the uninterfered signal layers of the virtual link, $\sum_{j=1}^{g_{bb}-\theta_0} L_{b,j}\left(\widehat{G}_b\right)\Big|_{g_{ab}=\theta_0}$. We bound the summation over j using a bound adapted from (3.57) coupled again with bounds of the form (A.163) and arrive at

$$\sum_{j=1}^{g_{bb}-\theta_0} L_{b,j}\left(\widehat{G}_b\right)\Big|_{g_{ab}=\theta_0} \leq \sum_{i=1}^{g_{aa}-(g_{bb}-\theta_0)} L_{a,i}\left(\widehat{G}_a\right)\Big|_{g_{ba}=g_{bb}-\theta_0} + n\left(g_{bb}-\theta_0\right) - nr_a\left(\widehat{G}_a\right)\Big|_{g_{ba}=g_{bb}-\theta_0} \tag{A.167}$$

$$\leq \sum_{i=1}^{g_{aa}-(g_{bb}-\theta_0)} L_{a,i}\left(\widehat{G}_a\right)\Big|_{g_{ba}=g_{bb}-\theta_0} + n\left(g_{bb}-\theta_0\right) - ng_{aa}\tau_a\left(g_{aa},g_{bb}\right) \tag{A.168}$$

$$\leq \sum_{i=1}^{\theta_1} L_{a,i}\left(\widehat{G}_a\right)\Big|_{g_{ba}=g_{bb}-\theta_0} + n\left(g_{aa}-\left(g_{bb}-\theta_0\right)-\theta_1\right)\tau_a\left(g_{aa},g_{bb}\right) + n\left(g_{bb}-\theta_0\right) - ng_{aa}\tau_a\left(g_{aa},g_{bb}\right) \tag{A.169}$$

$$= \sum_{i=1}^{\theta_1} L_{a,i}\left(\widehat{G}_a\right)\Big|_{g_{ba}=g_{bb}-\theta_0} - n\theta_1\tau_a\left(g_{aa},g_{bb}\right) + n\left(g_{bb}-\theta_0\right)\tau_b\left(g_{aa},g_{bb}\right) \tag{A.170}$$

where

$$\theta_1 = \left(g_{aa}-\left(g_{bb}-\theta_0\right)\right) \mod g_{bb} \tag{A.171}$$

$$= \left(g_{aa}+\theta_0\right) \mod g_{bb}. \tag{A.172}$$

If $\theta_1 = 0$, then we arrive a scenario like that in Figure A.3(b), where the second Link a (first virtual Link a) considered has a number of non-interfered signal layers that is evenly divisible by g_{bb}. If this is not the case, we may continue the growth of the virtual Z-channel and arrive at a bound on the remaining layers of the virtual

Link a:

$$\sum_{i=1}^{\theta_1} L_{a,i}\left(\widehat{G}_a\right)\Big|_{g_{ba}=g_{bb}-\theta_0} \leq \sum_{j=1}^{g_{bb}-\theta_1} L_{b,j}\left(\widehat{G}_b\right)\Big|_{g_{ab}=\theta_1} + n\theta_1 - n g_{bb}\tau_b\left(g_{aa}, g_{bb}\right) \tag{A.173}$$

$$\leq \sum_{j=1}^{g_{bb}-\theta_1} L_{b,j}\left(\widehat{G}_b\right)\Big|_{g_{ab}=\theta_1} - n\left(g_{bb}-\theta_1\right)\tau_b\left(g_{aa}, g_{bb}\right) + n\theta_1\tau_a\left(g_{aa}, g_{bb}\right), \tag{A.174}$$

and

$$\sum_{j=1}^{g_{bb}-\theta_2} L_{b,j}\left(\widehat{G}_b\right)\Big|_{g_{ab}=\theta_1} \leq \sum_{i=1}^{\theta_2} L_{a,i}\left(\widehat{G}_a\right)\Big|_{g_{ba}=g_{bb}-\theta_1} - n\theta_2\tau_a\left(g_{aa}, g_{bb}\right) + n\left(g_{bb}-\theta_2\right)\tau_b\left(g_{aa}, g_{bb}\right) \tag{A.175}$$

where

$$\theta_2 = \left(g_{aa}+\theta_1\right) \mod g_{bb}. \tag{A.176}$$

Though this process may seem cyclic, we note that

$$\theta_\ell = \left(g_{aa}+\ell\theta_0\right) \mod g_{bb}, \tag{A.177}$$

and that there exists some value for ℓ such that $\theta_\ell = 0$. When this is the case

$$\sum_{j=1}^{g_{bb}-\theta_{\ell-1}} L_{b,j}\left(\widehat{G}_b\right)\Big|_{g_{ab}=\theta_{\ell-1}} \leq n\theta_\ell\tau_a\left(g_{aa}, g_{bb}\right) - n\theta_\ell\tau_a\left(g_{aa}, g_{bb}\right) + n\left(g_{bb}-\theta_\ell\right)\tau_b\left(g_{aa}, g_{bb}\right) \tag{A.178}$$

$$= n\left(g_{bb}-\theta_{\ell-1}\right)\tau_b\left(g_{aa}, g_{bb}\right), \tag{A.179}$$

and

$$\sum_{i=1}^{\theta_1} L_{a,i}\left(\widehat{G}_a\right)\Big|_{g_{ba}=g_{bb}-\theta_0} \leq n\theta_{\ell-1}\tau_a\left(g_{aa},g_{bb}\right). \tag{A.180}$$

Carrying this process down to $\ell = 0$ and substituting into (A.166) yields

$$nr_a\left(\widehat{G}_a\right) \leq Ng_{aa}\tau_a\left(g_{aa},g_{bb}\right) \tag{A.181}$$

as desired.

To show that the input at Transmitter b is also constrained to its TDM allotted rate requires a single extension to the previously constructed Z-channel. If we now assume that the top link is a b-link and consider the possibility of $g_{ba} = g_{bb}$, then achievability of the TDM-like rate is reliant on

$$\begin{aligned}
nr_b\left(\widehat{G}_b\right) &\leq \sum_{j=1}^{g_{bb}} L_{b,j}\left(\widehat{G}_b\right) && \text{(A.182)}\\
&\leq \sum_{i=1}^{g_{aa}-g_{bb}} L_{a,i}\left(\widehat{G}_a\right)\Big|_{g_{ba}=g_{bb}} + ng_{bb} - nr_a\left(\widehat{G}_a\right)\Big|_{g_{ba}=g_{bb}} && \text{(A.183)}\\
&\leq n\left[\left(g_{aa}-g_{bb}\right)\tau_a\left(g_{aa},g_{bb}\right) + g_{bb} - r_a\left(\widehat{G}_a\right)\Big|_{g_{ba}=g_{bb}}\right] && \text{(A.184)}\\
&\leq n\left[\left(g_{aa}-g_{bb}\right)\tau_a\left(g_{aa},g_{bb}\right) + g_{bb} - g_{aa}\tau_a\left(g_{aa},g_{bb}\right)\right] && \text{(A.185)}\\
&= n\left[g_{bb} - g_{bb}\tau_a\left(g_{aa},g_{bb}\right)\right] && \text{(A.186)}\\
&= ng_{bb}\tau_b\left(g_{aa},g_{bb}\right). && \text{(A.187)}
\end{aligned}$$

as desired.

The statement for View 5 is a direct result of the result from View 3. In designing a policy let it be assumed that a genie will provide Transmitter a with knowledge of g_{ba}, and Transmitter b with knowledge of g_{ab}. The resulting genie-aided view is exactly the same as View 1, and thus the result that the capacity region is confined to that of TDM also holds.

□

LV-LDIC Views 4, 6, & 7

Proof of Theorem 7. We prove the results for Views 4, 6, & 7 by applying the results and intuitions gained from View 2. Specifically, whereas View 2 had two bottleneck cases — namely the channel state where the unknown direct link was equal to either interference gain — to prove the statement of Views 4, 6, & 7, we only require one worst case potential channel state for each: the case where the unknown links form a fully contested Z-channel. In the case of View 4 at Sender a, we apply the possibility that

$$g_{ab} = g_{bb} = g_{aa}, \tag{A.188}$$

which requires

$$r_a\left(\widehat{G}_a\right) + r_b\left(\widehat{G}_b\right)\Big|_{g_{ab}=g_{bb}=g_{aa}} \leq g_{aa}. \tag{A.189}$$

Notice that in order to satisfy the TDM condition, this inequality must be an equality which also implies

$$\frac{r_a\left(\widehat{G}_a\right)}{g_{aa}} \leq 1 - \frac{r_b(\widehat{G}_b)|_{g_{ab}=g_{bb}=g_{aa}}}{g_{aa}}. \tag{A.190}$$

We define

$$\tau_a^{g_{aa}} \triangleq \frac{r_a\left(\widehat{G}_a\right)}{g_{aa}}, \tag{A.191}$$

$$\tau_b^{g_{aa}} \triangleq \frac{r_b\left(\widehat{G}'_b\right)|_{g'_{ab}=g'_{bb}=g_{aa}}}{g_{aa}}, \tag{A.192}$$

where $\tau_a^{g_{aa}} + \tau_b^{g_{aa}} = 1$, and now consider the response of Sender b to its view of the channel. By considering an analogous Z-channel (where the direct link is fully interfered), we have

$$\tau_a^{g_{bb}} \triangleq \frac{r_a\left(\widehat{G}'_a\right)|_{g'_{ba}=g'_{aa}=g_{bb}}}{g_{bb}}, \tag{A.193}$$

$$\tau_b^{g_{bb}} \triangleq \frac{r_b\left(\widehat{G}_b\right)}{g_{bb}}, \tag{A.194}$$

where $\tau_a^{g_{bb}} + \tau_b^{g_{bb}} = 1$. The final step to completing the proof is to note that the TDM condition

$$\tau_a^{g_{aa}} + \tau_b^{g_{bb}} \geq 1, \tag{A.195}$$

for all G, requires that the inequality be an equality. Therefore the View 4 region is exactly that of TDM.

To demonstrate the theorem for Views 6 and 7, we need only consider the proper worst case Z-channels and apply the same logic. □

A.3 Gap between LV-LDIC and LV-GIC Capacity Regions

Proof of Theorem 8. In the process of bounding the gap between Gaussian and deterministic ICs, we assume satisfaction only of the more relaxed linear deterministic IC minimum performance criterion. We use heavily the result (3.186) from Section 3.4.2.

Additionally, we make the following observation

$$\sum_{\ell_1}^{\ell_2} \Lambda_a\left(\widehat{G}_a\right) - (\ell_2 - \ell_1)^+ \leq \log(3). \tag{A.196}$$

View 1

If $g_{aa} = g_{bb}$, then we refer the reader to the gap analysis for View 4, for which the worst case channel state assumes $g_{bb} = g_{aa}$. Otherwise, bounding the gap Δ_1 proceeds as follows.

We begin by manipulating expression (3.96) while noting (3.186) and arrive at a bound on the interference component of Transmitter a's signal.

$$\begin{aligned}\sum_{\ell=1}^{g_{ab}} \Lambda_{a,\ell}\left(\widehat{H}_a\right) \leq & \left(h(Y_b^n | W_{bb,u_b^+}, W_{ab,u_b^-}) - n\log(2\pi e) - I(X_b^n; Y_b^n)\right) \\ & + \left(\sum_{\ell=1}^{u_b^+} \Lambda_{b,\ell}\left(\widehat{H}_b'\right) + \sum_{\ell=1}^{u_b^-} \Lambda_{a,\ell}\right) \qquad \text{(A.197)} \\ \leq & \; n\min\left(g_{bb}, g_{ab}\right) + n\log(6) - nr_b\left(\widehat{H}_b'\right) \\ & + \sum_{\ell=1}^{u_b^+} \Lambda_{b,\ell}\left(\widehat{H}_b'\right) + \sum_{\ell=1}^{u_b^-} \Lambda_{a,\ell}\left(\widehat{H}_a\right). \qquad \text{(A.198)}\end{aligned}$$

Similarly, a constraint on the amount of interference in Transmitter b's signal is given by

$$\begin{aligned}\sum_{\ell=1}^{g_{ba}} \Lambda_{b,\ell}\left(\widehat{H}_b\right) \leq & \; n\min\left(g_{aa}, g_{ba}\right) + n\log(6) - nr_a\left(\widehat{H}_a'\right) \\ & + \sum_{\ell=1}^{u_a^+} \Lambda_{a,\ell}\left(\widehat{H}_a'\right) + \sum_{\ell=1}^{u_a^-} \Lambda_{b,\ell}\left(\widehat{H}_b\right). \qquad \text{(A.199)}\end{aligned}$$

In Appendix A.2, expressions analogous to (A.198) and (A.198) — namely (A.20) and (A.22) — were used to add virtual users representing different policy responses to a virtual Z-channel in constructing an outer bound for View 1 of the linear deterministic IC. As alluded to in Section 3.4, each additional interference event considered (each additional virtual user added to the virtual Z-channel) increases the gap between the linear deterministic and Gaussian outer bounds by log(6).

Adding additional virtual users accounts for any remaining sum of uninterfered signal layers (e.g., $\sum_{\ell=1}^{u_a^+} \Lambda_{a,\ell}\left(\widehat{H}_a'\right)$ in (A.199)). When a virtual user is not added (when the Z channel terminates), an additional gap between the Gaussian outer bound and its linear deterministic equivalent results from the quantization of the channel gain. For example

$$\sum_{\ell=1}^{u_a^+} \Lambda_{a,\ell}\left(\widehat{H}_a'\right) \leq \max_{p(X_a^n)} \sum_{\ell=1}^{u_a^+} \Lambda_{a,\ell}\left(\widehat{H}_a'\right) \leq n u_a^+ + \log(3). \tag{A.200}$$

In summary, the outer bounds constructed for View 1 in the Gaussian IC and linear deterministic IC have a gap that increases by log(6) per interferer considered, and by log(3) at the end of the Z chain (component bounds that are non-terminating lack the log(3) gap). We find gaps between the component bounds (3.106)–(3.119) and their Gaussian IC equivalents using this result, and detail them in Table A.1.

Outer Bound	**Component**	**Gap**	**Maximum ℓ**
(A.53)	r_a	$\log(3) + \log(6)$	—
(A.54)	r_b	$\log(3) + \log(6)$	—
(A.55)	$\overline{r}_a^c$	$\log(3) + (2\ell + 1)\log(6)$	$\left\lceil \frac{g_{bb}}{\delta} \right\rceil$
(A.56)	$\overline{r}_a^c$	$\log(3)$	—
(A.57)	$\overline{r}_b^c$	$\log(3) + \log(6)$	—
(A.58)	$\overline{r}_b^c$	$\log(3) + (2\ell + 1)\log(6)$	$\left\lceil \frac{g_{ba}}{\delta} \right\rceil$
(A.59)	$\overline{r}_a^c + r_b$	$\log(3) + (2\ell + 1)\log(6)$	$\left\lceil \frac{g_{bb}}{\delta} \right\rceil$
(A.60)	$\overline{r}_b^c + r_a$	$\log(3) + \log(6)$	—
(A.61)	$\overline{r}_b^c + r_b$	$\log(3) + (2\ell)\log(6)$	$\left\lceil \frac{g_{ba}}{\delta} \right\rceil - 1$
(A.62)	$\overline{r}_a^c + r_b - \overline{r}_b^c$	$(2\ell + 1)\log(6)$	$\left\lceil \frac{(g_{bb}-g_{ba})^+}{\delta} \right\rceil$
(A.63)	$\overline{r}_b^c + r_a - \overline{r}_a^c$	$(2\ell + 1)\log(6)$	$\left\lceil \frac{g_{ba}}{\delta} \right\rceil$
(A.64)	$r_a - \overline{r}_a^c$	$\log(3) + (2\ell + 1)\log(6)$	$\left\lceil \frac{g_{bb}}{\delta} \right\rceil$
(A.65)	$r_a - \overline{r}_a^c$	$\log(3) + \log(6)$	—
(A.66)	$r_b - \overline{r}_b^c$	$\log(3) + (2\ell - 1)\log(6)$	$\left\lceil \frac{(g_{bb}-g_{ba})^+}{\delta} \right\rceil$

Table A.1: Gap between component bounds (A.53)–(A.66) and Gaussian counterparts.

From Table A.1, and referring to the method of combining expressions (A.53)–(A.66), we also compute per-user gaps by combining the respective component bound gaps of Table A.1, and arrive at the bound in Table 3.1.

View 2

As in the linear deterministic IC version of View 2, two interference cases are sufficient to define a set of outer bounds. WLOG, we consider Transmitter a's response and mimic the derivation of bounds (3.120)–(3.123).

Following the derivation of (3.120) we have

$$
\begin{aligned}
nr_a\left(\widehat{G}_a\right) &\leq \max I(X_a^n; Y_a^n) && \text{(A.201)}\\
&\leq \max I(X_a^n; Y_a^n | X_b^n) && \text{(A.202)}\\
&\leq \max \sum_{i=1}^{g_{aa}} \Lambda_a\left(\widehat{G}_a\right) && \text{(A.203)}\\
&\leq n \max \log(1 + \|h_{aa}\|^2) && \text{(A.204)}\\
&\leq n \max \log(1 + 2^{g_{aa}+1}) && \text{(A.205)}\\
&\leq ng_{aa} + n\log(3). && \text{(A.206)}
\end{aligned}
$$

From the derivation of (3.121) we have

$$
\begin{aligned}
nr_a\left(\widehat{G}_a\right) &\leq I(X_a^n;Y_a^n) && \text{(A.207)}\\
&\leq \left(h(Y_a^n|W_{aa,u_a^+},W_{ba,u_a^-}) - n\log(2\pi e) - \sum_{\ell=1}^{g_{ba}}\Lambda_{b,\ell}\left(\widehat{G}_b\right)\right)\\
&\quad + \left(\sum_{\ell=1}^{u_a^+}\Lambda_{a,\ell}\left(\widehat{G}_a\right) + \sum_{\ell=1}^{u_a^-}\Lambda_{b,\ell}\left(\widehat{G}_b\right)\right) && \text{(A.208)}\\
&\leq \left(h(Y_a^n|W_{aa,u_a^+},W_{ba,u_a^-}) - n\log(2\pi e) - nr_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{ba}}\right)\\
&\quad + \left(\sum_{\ell=1}^{u_a^+}\Lambda_{a,\ell}\left(\widehat{G}_a\right) + \sum_{\ell=1}^{u_a^-}\Lambda_{b,\ell}\left(\widehat{G}_b\right)\right) && \text{(A.209)}\\
&\leq \big(h(Y_a^n|W_{aa,u_a^+},W_{ba,u_a^-}) - n\log(2\pi e) - ng_{ba}\tau_b\left(g_{ab},g_{ba}\right)\big)\\
&\quad + \left(\sum_{\ell=1}^{u_a^+}\Lambda_{a,\ell}\left(\widehat{G}_a\right) + \sum_{\ell=1}^{u_a^-}\Lambda_{b,\ell}\left(\widehat{G}_b\right)\right) && \text{(A.210)}\\
&\leq \left(n\log(6) + n\min\left(g_{aa},g_{ba}\right) - ng_{ba}\tau_b\left(g_{ab},g_{ba}\right)\right) && \text{(A.211)}\\
&\quad + \big(nu_a^+ + nu_a^- + n\log(3)\big) && \text{(A.212)}\\
&\leq n\left[g_{ba}\tau_a\left(g_{ab},g_{ba}\right) + \left(g_{aa}-g_{ba}\right)^+ + \log(6) + \log(3)\right]. && \text{(A.213)}
\end{aligned}
$$

If we consider Transmitter a's potential impact on Link b, we have

$$
\begin{aligned}
nr_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{ab}} &\leq I(X_b^n;Y_b^n) && \text{(A.214)}\\
&\leq \left(h(Y_b^n|W_{bb,u_b^+},W_{ab,u_b^-}) - n\log(2\pi e) - \sum_{\ell=1}^{g_{ab}}\Lambda_{a,\ell}\left(\widehat{G}_a\right)\right)\\
&\quad + \left(\sum_{\ell=1}^{u_b^+}\Lambda_{b,\ell}\left(\widehat{G}_b\right) + \sum_{\ell=1}^{u_b^-}\Lambda_{a,\ell}\left(\widehat{G}_a\right)\right) && \text{(A.215)}\\
&\leq ng_{ab} - \sum_{\ell=1}^{g_{ab}}\Lambda_{a,\ell}\left(\widehat{G}_a\right) + n\log(6), && \text{(A.216)}
\end{aligned}
$$

or

$$\sum_{\ell=1}^{g_{ab}} \Lambda_{a,\ell}\left(\widehat{G}_a\right) \leq n g_{ab} - n r_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{ab}} + n\log(6) \tag{A.217}$$

$$\leq n g_{ab} - n g_{ab}\tau_b\left(g_{ab}, g_{ba}\right) + n\log(6) \tag{A.218}$$

$$\leq n\left[g_{ab}\tau_a\left(g_{ab}, g_{ba}\right) + \log(6)\right], \tag{A.219}$$

which gives us

$$n r_a\left(\widehat{G}_a\right) \leq \max I(X_a^n; Y_a^n) \tag{A.220}$$

$$\leq \max I(X_a^n; Y_a^n | X_b^n) \tag{A.221}$$

$$\leq \max \sum_{i=1}^{g_{aa}} \Lambda_a\left(\widehat{G}_a\right) \tag{A.222}$$

$$\leq \max \sum_{i=1}^{g_{ab}} \Lambda_a\left(\widehat{G}_a\right) + \sum_{i=g_{ab}+1}^{g_{aa}} \Lambda_a\left(\widehat{G}_a\right) \tag{A.223}$$

$$\leq n\left[g_{ab}\tau_a\left(g_{ab}, g_{ba}\right) + \log(6) + \left(g_{aa} - g_{ab}\right)^+ + \log(3)\right], \tag{A.224}$$

and

$$
\begin{aligned}
nr_a\left(\widehat{G}_a\right) &\leq \max I(X_a^n;Y_a^n) && \text{(A.225)}\\
&\leq \max I(X_a^n;Y_a^n|X_b^n) && \text{(A.226)}\\
&\leq \max \sum_{i=1}^{g_{aa}} \Lambda_a\left(\widehat{G}_a\right) && \text{(A.227)}\\
&\leq \max \sum_{i=1}^{g_{ab}} \Lambda_a\left(\widehat{G}_a\right) + \sum_{i=g_{ab}+1}^{g_{aa}} \Lambda_a\left(\widehat{G}_a\right) && \text{(A.228)}\\
&\leq \max n g_{ab}\tau_a\left(g_{ab},g_{ba}\right) + n\log(6)\\
&\quad + \sum_{i=g_{ab}+1}^{g_{aa}-g_{ba}} \Lambda_a\left(\widehat{G}_a\right) + \sum_{i=g_{aa}-g_{ba}+1}^{g_{aa}} \Lambda_a\left(\widehat{G}_a\right), && \text{(A.229)}\\
&\leq n\left[g_{ab}\tau_a\left(g_{ab},g_{ba}\right) + \log(6) + \left(g_{aa}-g_{ab}-g_{ba}\right)^+ + \log(3)\right.\\
&\quad \left. + g_{ba}\tau_a\left(g_{ab},g_{ba}\right) + \log(6)\right]. && \text{(A.230)}
\end{aligned}
$$

Consequently, we bound the Link a capacity gap (and Link b by parallel analysis) comparing (A.206), (A.213), (A.224), and (A.230) to (3.120)–(3.123), and arrive at the stated bound.

Views 3 & 5

Proof of Views 3 and 5 relies on bounding disjoint sets of g_{bb} consecutive layers of Transmitter a's input with the expression (A.163). In the Gaussian IC version, by applying (3.186) we have

$$
\sum_{i=\kappa+1}^{\kappa+g_{bb}} \Lambda_{a,i}\left(\widehat{G}_a\right) \leq n\left(g_{bb}\tau_a(g_{aa},g_{bb}) + \log(6)\right). \qquad \text{(A.231)}
$$

To bound the total per-user gap to the Gaussian IC capacity region we track how the gap accumulates in the construction of the virtual Z-channel used in the linear deterministic proof. Let $\tilde{\Delta}_3[\ell]$ be the total gap after ℓ virtual Link as have been considered. When $\ell = 1$, we find

$$\tilde{\Delta}_3[1] = \left(\frac{g_{aa} - \theta_0}{g_{bb}}\right) \log(6). \tag{A.232}$$

Recall that if $\theta_0 = 0$, then g_{aa} was evenly divisible by g_{bb} and the proof ends. If $\theta_0 \neq 0$ we can find through induction

$$\tilde{\Delta}_3[\ell] = \left(\tilde{\Delta}_3[\ell - 1] + 2 + \frac{g_{aa} - \theta_\ell - (g_{bb} - \theta_\ell - 1)}{g_{bb}}\right) \log(6) \tag{A.233}$$

$$= \left(\frac{\ell g_{aa} - \theta_{\ell-1}}{g_{bb}} + \ell - 1\right) \log(6). \tag{A.234}$$

When $\theta_{\ell-1} = 0$, the chain of substitutions in the proof of Theorem 6 ends, implying ℓg_{aa} is evenly divisible by g_{bb}. Consequently, if we let $\ell^\star$ be the minimum value of ℓ where this occurs, $\ell^\star g_{aa}$ is the least common multiple of g_{aa} and g_{bb}, and it becomes clear that

$$\Delta_3 = \left(\frac{\ell^\star g_{aa}}{g_{bb}} + \ell^\star - 1\right) \log(6) \tag{A.235}$$

$$= \left(\frac{\ell^\star g_{aa}}{g_{bb}} + \frac{\ell^\star g_{aa}}{g_{aa}} - 1\right) \log(6) \tag{A.236}$$

$$= \left(\frac{\mathrm{LCM}(g_{aa}, g_{bb})}{g_{bb}} + \frac{\mathrm{LCM}(g_{aa}, g_{bb})}{g_{aa}} - 1\right) \log(6). \tag{A.237}$$

Views 4, 6, & 7

For View 4, when Transmitter a considers the case $g_{ba} = g_{bb} = g_{aa}$

$$\begin{aligned}
nr_a\left(\widehat{G}_a\right) &\leq I(X_a^n; Y_a^n) && \text{(A.238)}\\
&\leq \left(h(Y_a^n | W_{aa,u_a^+}, W_{ba,u_a^-}) - n\log(2\pi e) - \sum_{\ell=1}^{g_{ba}} \Lambda_{b,\ell}\left(\widehat{G}_b\right)\right)\\
&\quad + \left(\sum_{\ell=1}^{u_a^+} \Lambda_{a,\ell}\left(\widehat{G}_a\right) + \sum_{\ell=1}^{u_a^-} \Lambda_{b,\ell}\left(\widehat{G}_b\right)\right) && \text{(A.239)}\\
&\leq \left(n\log(6) + n\min\left(g_{aa}, g_{ba}\right) - nr_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{aa}}\right) && \text{(A.240)}\\
&\leq \left(n\log(6) + ng_{aa} - ng_{aa}\tau_b\right) && \text{(A.241)}\\
&\leq n\left[g_{aa}\tau_a + \log(6)\right]. && \text{(A.242)}
\end{aligned}$$

For View 6, consider the case $g_{ab} = g_{bb} = g_{aa}$

$$\begin{aligned}
nr_a\left(\widehat{G}_a\right) &\leq I(X_a^n; Y_a^n) && \text{(A.243)}\\
&\leq \sum_{\ell=1}^{g_{aa}} \Lambda_{a,\ell}\left(\widehat{G}_a\right) && \text{(A.244)}\\
&\leq ng_{aa} - nr_b\left(\widehat{G}_b\right)\Big|_{g_{bb}=g_{aa}} + n\log(6) && \text{(A.245)}\\
&\leq ng_{aa} - ng_{aa}\tau_b + n\log(6) && \text{(A.246)}\\
&\leq n\left[g_{aa}\tau_a + \log(6)\right]. && \text{(A.247)}
\end{aligned}$$

The analysis for View 7 may follow that of either View 4 or View 6, and comparison of the resulting expression with the linear deterministic analogue confirms the stated claim.

□

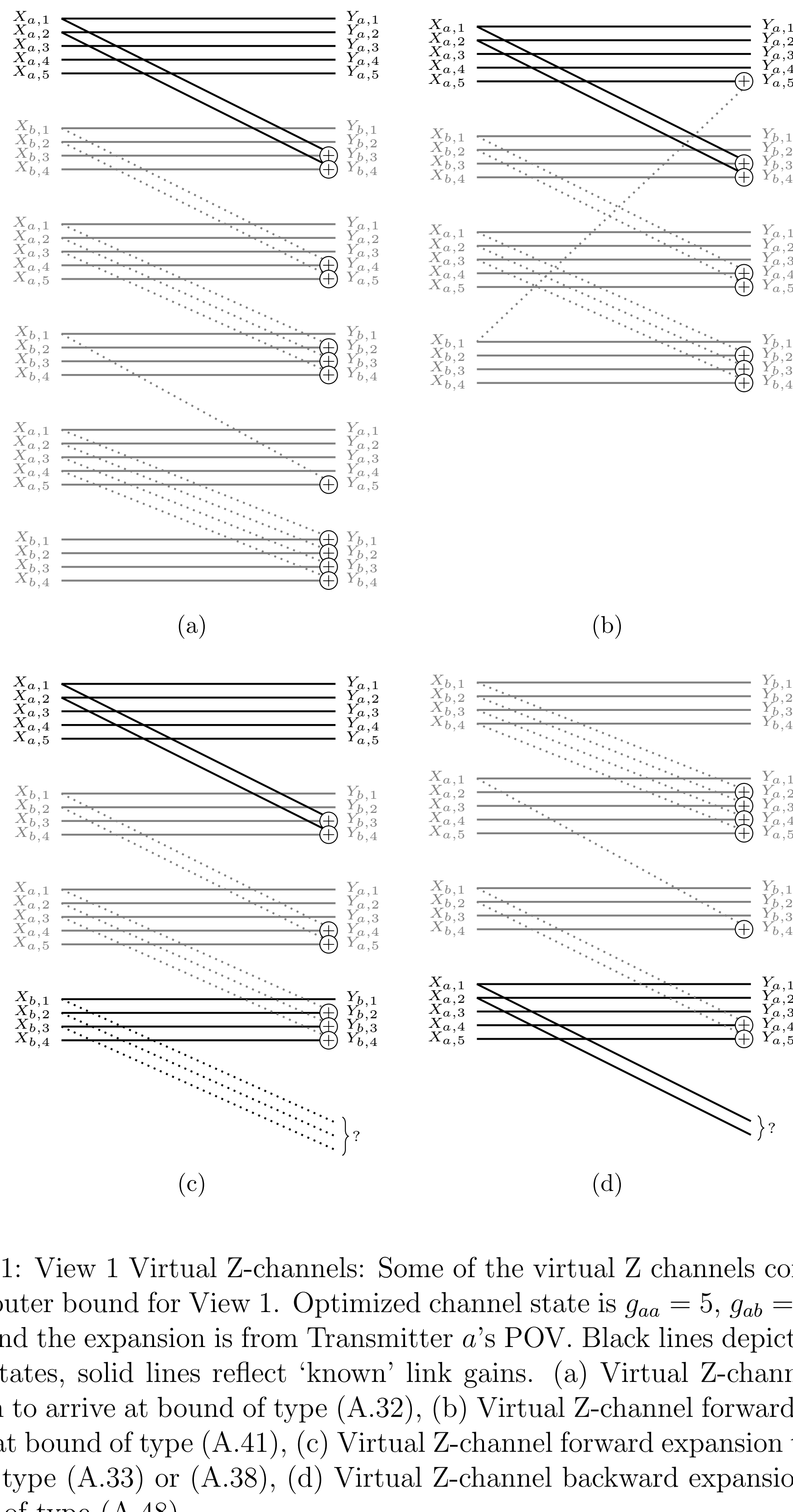

Figure A.1: View 1 Virtual Z-channels: Some of the virtual Z channels considered in deriving outer bound for View 1. Optimized channel state is $g_{aa} = 5$, $g_{ab} = 2$, $g_{ba} = 3$, $g_{bb} = 4$, and the expansion is from Transmitter a's POV. Black lines depict optimized channel states, solid lines reflect 'known' link gains. (a) Virtual Z-channel forward expansion to arrive at bound of type (A.32), (b) Virtual Z-channel forward expansion to arrive at bound of type (A.41), (c) Virtual Z-channel forward expansion to arrive at bound of type (A.33) or (A.38), (d) Virtual Z-channel backward expansion to arrive at bound of type (A.48).

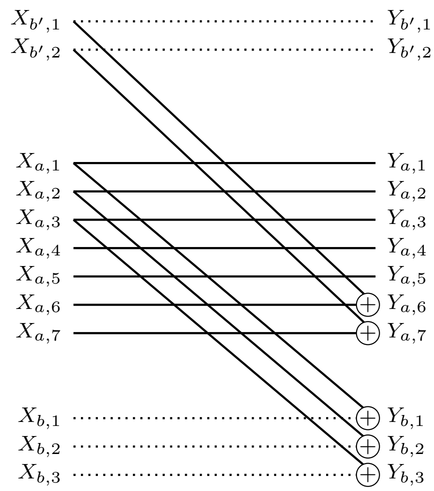

Figure A.2: View 2 Virtual Z-channel: Virtual double Z-channel for Transmitter a's View 2. Dotted segments represent unknown link gains.

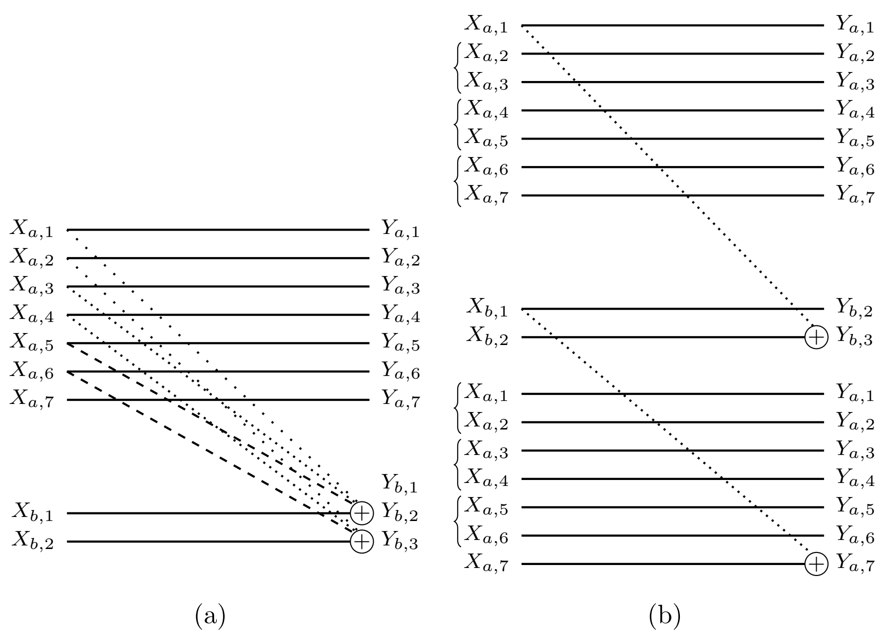

Figure A.3: View 3 Virtual Z-channels: Virtual Z channels used to derive outer bound for View 3: (a) Some of the two-user virtual Z channels used to derive (A.158)–(A.161), and (b) larger virtual Z to bound relatively prime direct link channels. Bracketed inputs are bounded by constraints from (A.158)–(A.161) and other inputs are constrained by potential interference interactions.

A.4 Local View Gaussian Interference Channel with Receiver Cooperation

All analyses are based on a parameterization of the set of policies. We let $\tau \in [0,1]$ parameterize the baseline TDM policy, and consider only those policies where the resulting rate pair is a Pareto improvement over the TDM rate pair.

Joint Decoding Lemma

The SIMO-MAC simulated by receiver cooperation permits a powerful decoding architecture known as joint decoding. In joint decoding, the codewords of both transmitters are simultaneously decoded (as opposed to sequentially), and as a result, the design of codebooks robust to unknown state need only rely on the joint typicality of codewords.

Lemma 35 ([18]). *A policy with codebooks (satisfying typicality criteria) drawn from independently distributed Gaussian distributions is achievable as long as for all possible channel gains the codebook rates satisfy*

$$r_a\left(\widehat{H}_a\right) \leq \log\left(1 + |h_{aa}|^2 + |h_{ab}|^2 + 2\,|h_{aa}|\,|h_{ab}|\right) \tag{A.248}$$

$$r_b\left(\widehat{H}_b\right) \leq \log\left(1 + |h_{bb}|^2 + |h_{ba}|^2 + 2\,|h_{bb}|\,|h_{ba}|\right) \tag{A.249}$$

$$r_a\left(\widehat{H}_a\right) + r_b\left(\widehat{H}_b\right) \leq \log\det\left(\mathbf{I} + \mathbf{H}\mathbf{H}^{\dagger}\right), \tag{A.250}$$

for all channels H resulting in local views $\widehat{H}_a$ and $\widehat{H}_b$

Consequently, in analyzing the capacity regions of the LV-GIC with receiver cooperation, we omit demonstrating of achievability for each case.

Views 1 & 2

Proof of Theorem 10. As mentioned in the body of Chapter 4, expressions (4.13)–(4.15) are exactly the full-view capacity region. We arrive at (4.16) by considering the sum-rate bound (4.9) and all possible views seen at the other transmitter, we consider cases where $\angle h'_{ba} = \angle(h_{aa}h^*_{ab}h_{bb}) + \pi$ and find that r_a must satisfy

$$\begin{aligned} r_a \leq \; & \log\Big(1 + |h_{aa}|^2 + |h_{ab}|^2 + |h'_{ba}|^2 + |h_{bb}|^2 \\ & + |h_{aa}|^2 |h_{bb}|^2 + |h_{ab}|^2 |h'_{ba}|^2 - 2\,|h_{aa}|\,|h_{ab}|\,|h'_{ba}|\,|h_{bb}|\Big) - r'_b, \end{aligned} \tag{A.251}$$

for all values of $|h'_{ba}|$. By application of the receiver beamforming-based MPC described in §4.1 and minimizing over all h'_{ba}, we arrive at (4.16).

An analogous analysis yield (4.17). □

View 3

Proof of Theorem 12. For View 3, we first note that receive beamforming may have no benefit since it is possible that the parameter unknown to Transmitter a or b (h_{ab} or h_{ba} respectively) may be zero. This provides us with the single user capacity bounds (4.22) and (4.23). The expression (4.24) is the sum-rate bound for the channel state being considered.

For (4.25), we consider cases where $\angle h'_{ab} = \angle(h_{aa}h^*_{ba}h_{bb}) + \pi$, and fix $r'_b = \tau \log\left(1 + |h_{bb}|^2\right)$. We minimize the sum-rate bound (4.9) over possible values of h'_{ab} to find

$$\begin{aligned} |h^\star_{ab}| = \; & \underset{s\in\mathbb{R}^+}{\operatorname{argmin}} \log\big(1 + |h_{aa}|^2 + s^2 + |h_{ba}|^2 + |h_{bb}|^2 \\ & + |h_{aa}|^2 |h_{bb}|^2 + |h_{ba}|^2 s^2 - 2\,|h_{aa}|\,|h_{ba}|\,|h_{bb}|\,s\big) \end{aligned} \tag{A.252}$$

$$= \frac{|h_{aa}|\,|h_{ba}|\,|h_{bb}|}{1 + |h_{ba}|^2}. \tag{A.253}$$

Substitution of this potential channel gain into the sum rate expression, along with the parameterization of MPC rate of Transmitter b yields (4.25).

An analogous analysis yield (4.26). $\square$

View 4

Proof of Theorem 14. The region given for View 4 results directly from the region given for View 1, while also minimizing over the unknown direct link of the other user. $\square$

Proof of Corollary 15. There exists a possibility that the unknown parameters for Transmitter a are such that $h'_{ba} = h_{aa}$ and $h'_{bb} = h_{ab}$. This results in the following sum-rate bound

$$
\begin{aligned}
r_a \leq\ & \log\left(1 + 2\left|h_{aa}\right|^2 + 2\left|h_{ab}\right|^2\right) - r'_b && \text{(A.254)}\\
\leq\ & \log\left(1 + 2\left|h_{aa}\right|^2 + 2\left|h_{ab}\right|^2\right) - \tau\log\left(1 + \left|h_{aa}\right|^2 + 2\left|h_{aa}\right|\left|h_{ab}\right| + \left|h_{ab}\right|^2\right) && \text{(A.255)}\\
=\ & \log\left(\frac{1 + \left(\left|h_{aa}\right| + \left|h_{ab}\right|\right)^2 + \left(\left|h_{aa}\right| - \left|h_{ab}\right|\right)^2}{1 + \left(\left|h_{aa}\right| + \left|h_{ab}\right|\right)^2}\right)\\
& + (1-\tau)\log\left(1 + \left|h_{aa}\right|^2 + 2\left|h_{aa}\right|\left|h_{ab}\right| + \left|h_{ab}\right|^2\right) && \text{(A.256)}\\
\leq\ & \log(2) + (1-\tau)\log\left(1 + \left|h_{aa}\right|^2 + 2\left|h_{aa}\right|\left|h_{ab}\right| + \left|h_{ab}\right|^2\right). && \text{(A.257)}
\end{aligned}
$$

The term $(1-\tau)\log\left(1 + \left|h_{aa}\right|^2 + 2\left|h_{aa}\right|\left|h_{ab}\right| + \left|h_{ab}\right|^2\right)$ is the enhanced TDM rate for Transmitter a, and thus we have at most one bit extra for Transmitter a's rate. Similar analysis holds for Transmitter b. $\square$

View 5

Proof of Theorem 17. We first note that the single user capacity bounds (4.40) and (4.41) result from the possibility of $h_{ab} = h_{ba} = 0$.

Now, consider the SIMO-MAC sum-rate boundary (4.9). First, we note that the expression (4.9) is minimized for fixed magnitude h'_{ab} and h'_{ba} when $\angle\left(h'_{ab}h'_{ba}\right) = \angle\left(h_{aa}h_{bb}\right) + \pi$. We then note that the resulting operand of the logarithm,

$$1 + |h_{aa}|^2 + |h'_{ab}|^2 + |h'_{ba}|^2 + |h_{bb}|^2 + |h_{aa}|^2 |h_{bb}|^2 + |h'_{ab}|^2 |h'_{ba}|^2 - 2 |h_{aa}| |h'_{ab}| |h'_{ba}| |h_{bb}| , \tag{A.258}$$

is quadratic and convex with respect to $|h'_{ab}|$ and $|h'_{ba}|$ and by minimizing over positive real magnitudes for $|h'_{ab}|$ and $|h'_{ba}|$ we arrive at minimizers

$$|h^{\star}_{ab}| = |h^{\star}_{ba}| = \sqrt{|h_{aa}| |h_{bb}|}. \tag{A.259}$$

Notice that the resulting channel matrix

$$\mathbf{H}^{\star} = \begin{bmatrix} h_{aa} & h^{\star}_{ba} \\ h^{\star}_{ab} & h_{bb} \end{bmatrix}, \tag{A.260}$$

is rank deficient, and the resulting sum rate bound is given by (4.42). □

View 6

Proof of Theorem 19. Expressions (4.51)–(4.53) result from the same logic as in the proof of Theorem 12. To arrive at (4.54) we consider the expression (4.25) and minimize over potential values of h_{bb}.

Notice that

$$\log\left(1 + s + |h_{ba}|^2\right) - \tau \log\left(1 + s\right) \tag{A.261}$$

is quasi-convex in s and is minimized when

$$s^\star = \frac{\tau \left|h_{ba}\right|^2 - (1-\tau)}{1-\tau}. \tag{A.262}$$

Substituting $h_{bb} = s^\star$ into (4.25) yields (4.54).

Similarly we can arrive at (4.55) from (4.26). □

Proof of Corollary 20. We first note that (4.49) may be rewritten as

$$r_a \leq \max\left\{\log\left(\frac{1+\left|h_{aa}\right|^2}{1+\left|h_{ba}\right|^2}\right), 0\right\} + (1-\tau)\log\left(1+\left|h_{ba}\right|^2\right). \tag{A.263}$$

Now, consider the gap between (4.54) and (4.49). If $|h_{aa}| \geq |h_{ba}|$:

$$\begin{aligned}
\Delta =\ & \log\left(1+\frac{\left|h_{aa}\right|^2}{1+\left|h_{ba}\right|^2}\right) + (1-\tau)\log\left(\left|h_{ba}\right|^2\right) + H_0(\tau) \\
& - \log\left(\frac{1+\left|h_{aa}\right|^2}{1+\left|h_{ba}\right|^2}\right) - (1-\tau)\log\left(1+\left|h_{ba}\right|^2\right) && \text{(A.264)} \\
=\ & \log\left(1+\frac{\left|h_{ba}\right|^2}{1+\left|h_{aa}\right|^2}\right) + H_0(\tau) && \text{(A.265)} \\
\leq\ & 2. && \text{(A.266)}
\end{aligned}$$

If $|h_{aa}| < |h_{ba}|$:

$$\begin{aligned}
\Delta =\ & \log\left(1+\frac{\left|h_{aa}\right|^2}{1+\left|h_{ba}\right|^2}\right) + (1-\tau)\log\left(\left|h_{ba}\right|^2\right) + H_0(\tau) - (1-\tau)\log\left(1+\left|h_{ba}\right|^2\right) \\
& && \text{(A.267)} \\
=\ & \log\left(1+\frac{\left|h_{aa}\right|^2}{1+\left|h_{ba}\right|^2}\right) + H_0(\tau) && \text{(A.268)} \\
\leq\ & 2. && \text{(A.269)}
\end{aligned}$$

□

View 7

Proof of Theorem 22. We begin with the result of View 5 with Receiver cooperation and minimize the expression (4.42) over possible values of h_{bb} for Transmitter a's local view to arrive at (4.63). Similarly, minimizing over possible values of h_{aa} yields the constraint on Transmitter b (4.64). □

Proof of Corollary 23. Consider the bound (4.63) and the parameterized rate achieved through TDM $r_a = (1-\tau)\log\left(1+|h_{aa}|^2\right)$. The gap between the TDM achievable rate and capacity can be bounded as

$$\begin{aligned}
\Delta = & \log\left(1+|h_{aa}|^2+|h_{bb}^{\star}|^2+2\,|h_{aa}|\,|h_{bb}^{\star}|\right)-\tau\log\left(1+|h_{bb}^{\star}|^2\right) \\
& -(1-\tau)\log\left(1+|h_{aa}|^2\right) && \text{(A.270)} \\
\leq & \log\left(1+|h_{aa}|^2+|h_{aa}|^2+2\,|h_{aa}|\,|h_{aa}|\right)-\tau\log\left(1+|h_{aa}|^2\right) \\
& -(1-\tau)\log\left(1+|h_{aa}|^2\right) && \text{(A.271)} \\
\leq & \log\left(1+4\,|h_{aa}|^2\right)-\log\left(1+|h_{aa}|^2\right) && \text{(A.272)} \\
\leq & \, 2. && \text{(A.273)}
\end{aligned}$$

□

A.5 Local View Gaussian Interference Channel with Transmitter Cooperation

View 1

Proof of Proposition 26. For View 1, we note that it is possible that the parameter unknown to Transmitter a or b (h_{ba} or h_{ab} respectively) may be zero. Consequently, we arrive at the single-user bounds (5.12) and (5.13). The bound (5.12) is the sum-rate bound of the channel considered.

Because the parameters enabling transmit beamforming are unknown, we apply the MPC associated with $\mathcal{R}_G^{\mathsf{TDM}}$, and consider the satisfaction of (5.8) for channel states other than the one considered.

For (5.15), we assume $r'_b = \log(1 + |h_{bb}|^2)$, and minimize (5.8) over possible values of h_{ba}. We use the fact that the operand of the logarithm is quadratic (and thus convex) with respect to h_{ba}, and the analysis is similar to that of the proof of Theorem 12 (for brevity we omit the steps here). An analogous analysis yield (5.16). □

Views 2 & 3

Proof of Proposition 27. The outer bound presented in Proposition 27 simply consists of a set of the full view MISO-BC outer bounds (5.17)–(5.19) and a minimization of the sum-rate outer bound (5.8) minimized over the unknown parameter h_{ab}. We note that the resulting expressions (5.20) and (5.21) exist not to guarantee achievability of the coding scheme for the channel state considered, but rather are necessary conditions for satisfaction of the minimum performance criterion. □

View 4

Proof of Proposition 28. The outer bound presented in Proposition 28 is a direct application of the bounds from View 1 with message-only transmit cooperation. Note that the only difference lies in the expressions (5.25) and (5.26), where we have defined each bound as a minimization over the additional unknown parameter (h_{bb} for Transmitter a and h_{aa} for Transmitter b). □

View 5

Proof of Lemma 29. The approach taken relies on joint decoding of a jointly encoded Message b at both receivers and successive decoding of Message a at Receiver a.

We first consider conditions for decodability of Message b at Receiver b. Note that output of the channel at Receiver b is

$$Y_b = h_{bb}X_{b,1} + h_{ab}X_{b,2} + h_{ab}X_a + Z_b. \tag{A.274}$$

Receiver b can decode its message, treating the message of a as noise if

$$\log\left(1 + \frac{\mathrm{var}(h_{bb}X_{b,1} + h'_{ab}X_{b,2})}{1 + \mathrm{var}(h'_{ab}X_a)}\right) = \log\left(1 + \frac{|h_{bb}|^2 + s|h'_{ab}|^2}{1 + (1-s)|h'_{ab}|^2}\right) \tag{A.275}$$

$$\geq r_b, \tag{A.276}$$

for all h'_{ab}. Notice that the term $\frac{|h_{bb}|^2+s|h'_{ab}|^2}{1+(1-s)|h'_{ab}|^2}$ is monotone with respect to $|h'_{ab}|$, and therefore the bound $\log\left(1 + \frac{|h_{bb}|^2+s|h'_{ab}|^2}{1+(1-s)|h'_{ab}|^2}\right)$ is minimized either when $|h'_{ab}| = 0$ or as $|h'_{ab}| \to \infty$. The two terms in the min governing the rate of Codebook b are either as tight or tighter, so Message b is decodable.

Now consider decodability of Message b at Receiver a; the message is decodable treating Message a as noise if

$$\log\left(1+\frac{\mathrm{var}(h'_{ba}X_{b,1}+h_{aa}X_{b,2})}{1+\mathrm{var}(h_{aa}X_a)}\right) = \log\left(1+\frac{|h'_{ba}|^2+s|h_{aa}|^2}{1+(1-s)|h_{aa}|^2}\right) \tag{A.277}$$

$$\geq r_b, \tag{A.278}$$

for all h'_{ba}. This bound is minimized when $h'_{ba}=0$, however the expression (5.31) has ensured the rate of Codebook b satisfies this scenario.

Finally, after Message b has been decoded at Receiver a, Receiver a removes the signal component associated with Message b and decodes the remaining signal as it were a single-user channel. □

Proof of Proposition 30. The outer bound presented in Proposition 30 results from two possible scenarios. First, if $h'_{ab}=h'_{ba}=0$ then we arrive at the single-user capacities given by (5.32) and (5.33).

To arrive at (5.34), we consider the bound (5.8) and consider the case where $h'_{ab}=h_{aa}$ and $h'_{ba}=h_{bb}$. □

Proof of Theorem 31. To prove Theorem 31, we note first that the single-user capacities can be achieved by allowing s to be either zero or one respectively. To prove that the sum rate achieved by our scheme is close to the sum-rate outer bound, we loosen the outer bound (5.34) in the following manner (we assume as before and WLOG that $|h_{aa}|\leq|h_{bb}|$)

$$\log\left(1+2\left(|h_{aa}|^2+|h_{bb}|^2+2\,|h_{aa}|\,|h_{bb}|\right)\right)$$

$$\leq \log\left(1+2\left(|h_{aa}|^2+|h_{aa}|^2+2\,|h_{aa}|\,|h_{aa}|\right)\right) \tag{A.279}$$

$$= \log\left(1+8\,|h_{aa}|^2\right). \tag{A.280}$$

Noting that the region achieved by our policy may be rewritten as

$$r_a \leq \log\left(1 + |h_{aa}|^2\right) \tag{A.281}$$

$$r_b \leq \log\left(1 + |h_{bb}|^2\right) \tag{A.282}$$

$$r_a + r_b \leq \log\left(1 + |h_{aa}|^2\right), \tag{A.283}$$

We find that the gap can be bounded as

$$\Delta \leq \log\left(1 + 8|h_{aa}|^2\right) - \log\left(1 + |h_{aa}|^2\right) \tag{A.284}$$

$$= \log\left(\frac{1 + 8|h_{aa}|^2}{1 + |h_{aa}|^2}\right) \tag{A.285}$$

$$\leq 3. \tag{A.286}$$

□

View 6

Proof of Theorem 32. Consider the scenario where $h_{aa} = h_{ab}$ and $h_{ba} = h_{bb}$. In this scenario, the received signals at Receiver a and b are identical with the exception of the noise component. With respect to decodability of codewords they are the same: any codeword decodable at Receiver a with error rate vanishing with block length is similarly decodable with vanishing error at Receiver b. Consequently, in this channel state the messages intended for either receiver is either decodable by both or undecodable, and the single-user capacity achievable by transmitter beamforming must be split between two messages.

In the case of View 6, although the channel state may not be as described above, each transmitter must accommodate this possibility, and in the absence of additional common knowledge, the splitting of single-user capacity must be fixed. As a result, neither transmitter may achieve better than a TDM-like splitting of transmitter beamforming rate associated with $\mathcal{R}_{GT}^{\mathsf{TDM-E}}$. □

View 7

Proof of Theorem 33. From the minimum performance criterion associated with $\mathcal{R}_{G}^{\mathsf{TDM}}$ we assume $r_b' \geq \tau \log(1 + |h_{bb}'|^2)$ and consider the case where $h_{aa} = h_{ab} = h_{ba} = h_{bb}$. From (5.8), we can express a bound on the sum rate of this scenario as

$$
\begin{aligned}
r_a &\leq \log\left(1 + 8|h_{aa}|^2\right) - r_b' && \text{(A.287)}\\
&\leq \log\left(1 + 8|h_{aa}|^2\right) - \tau \log\left(1 + |h_{aa}|^2\right) && \text{(A.288)}\\
&\leq \log\left(\frac{1 + 8|h_{aa}|^2}{1 + |h_{aa}|^2}\right) + (1 - \tau) \log\left(1 + |h_{aa}|^2\right) && \text{(A.289)}\\
&\leq 3 + (1 - \tau) \log\left(1 + |h_{aa}|^2\right). && \text{(A.290)}
\end{aligned}
$$

Notice that the rightmost quantity of (A.290) is the parameterized TDM rate for Transmitter a, and that the gain over this rate is bounded by 3 as desired.

□